Proofs of formal claims in review of *Arguing About Alliances*
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Two states $i = \{A, B\}$ negotiate over where to set a policy x on the real number line. Each has an ideal policy, or an ideal war plan, x_i, where without loss of generality $x_A < x_B$. Let players consider policies only $x \in [x_A, x_B]$, such that we can write their payoffs for some x as linear loss functions $u_A(x) = x_A - x$ and $u_B(x) = x - x_B$. Ideal points are commonly known, but outside options $u_i(\text{na}) = -d_i$ are private information, where the costs of nonagreement d_i are distributed according to $d_i \sim \text{U}(\underline{d}_i, \overline{d}_i)$.

Suppose that A makes a take-it-or-leave-it (TILI) proposal, which B accepts iff

$$x - x_B \geq -d_B \Leftrightarrow d_B \geq x_B - x = \hat{d}_B,$$

or when its outside option is sufficiently unattractive. A's expected utility for proposing x is

$$EU_A(x) = \left(\frac{\hat{d}_B}{\overline{d}_B - \underline{d}_B}\right)\left(-d_A\right) + \left(\frac{\overline{d}_B - \hat{d}_B}{\overline{d}_B - \underline{d}_B}\right)\left(x_A - x\right),$$

which it maximizes at

$$x^* = \frac{x_A + x_B + d_A - \overline{d}_B}{2}.$$

This lets us write the probability of agreement as

$$\Pr(d_B < \hat{d}_B | x^*) = \frac{\overline{d}_B - (x_B - x^*)}{\overline{d}_B - \underline{d}_B},$$

which after substitution yields

$$\Pr(\text{agreement}) = \frac{d_A + \overline{d}_B - (x_B - x_A)}{2\left(\overline{d}_B - \underline{d}_B\right)}.$$

Note that as the difference $(x_B - x_A)$ shrinks—i.e., as commonly known war plans grow more compatible—the probability of agreement increases, even as states remain uncertain over the attractiveness of outside options. Likewise, the effects of uncertainty, which we can take as $(\overline{d}_B - \underline{d}_B)$, depend on the divergences between ideal points.