Showing Restraint, Signaling Resolve: Supporting Information

Scott Wolford
University of Texas
swolford@austin.utexas.edu

July 27, 2013

This online appendix supplements:

Wolford, Scott. Forthcoming. "Showing Restraint, Signaling Resolve: Coalitions, Cooperation, and Crisis Bargaining." *American Journal of Political Science*.

Proof of Proposition 1. I first prove the existence of separating and semi-separating equilibria, then show that pure pooling equilibria, on both high and low mobilizations, do not exist.

The separating equilibrium occurs when $m_T \geq \hat{m}_T$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. T: believe $\phi' = 1$ if $m_L^* = \overline{m}_L$ and propose $x^* = \overline{x}$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$ and propose $x^* = \underline{x}$. Since each possible mobilization level is chosen, there are no out-of-equilibrium beliefs.

To prove the existence of this equilibrium, we first derive each type of L's optimal mobilization level given its type. The resolute L chooses $\overline{m}_L$ such that it satisfies

$$\max_{m_L}\{-m_L + \frac{m_L}{m_L + m_T}\overline{v}_L - c_L\}.$$

The optimum satisfies the first-order condition, $m_T\overline{v}_L/(m_T + m_L)^2 - 1 = 0$, yielding

$$\overline{m}_L = \sqrt{m_T}\sqrt{\overline{v}_L} - m_T. \tag{1}$$

Substituting $\overline{m}_L$ into the second partial with respect to m_L yields $-2/\left(\sqrt{m_T}\sqrt{\overline{v}_L}\right) < 0$, ensuring that $\overline{m}_L$ is a maximum. Since player-types' payoffs differ only in their valuations v_L, we can also define the irresolute type's optimum as

$$\underline{m}_L = \sqrt{m_T}\sqrt{\underline{v}_L} - m_T,$$

where $\underline{m}_L < \overline{m}_L$, since $\underline{v}_L < \overline{v}_L$. Next, consider what proposals L accepts after choosing m_L. The resolute type accepts iff x satisfies

$$x\overline{v}_L \geq \frac{\overline{m}_L}{\overline{m}_L + m_T}\overline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{c_L + \sqrt{m_T}\sqrt{\overline{v}_L}}{\overline{v}_L} \equiv \overline{x}, \tag{2}$$

and the irresolute type's range of acceptable proposals can be defined by the same process as

$$x \geq 1 - \frac{c_L + \sqrt{m_T}\sqrt{\underline{v}_L}}{\underline{v}_L} \equiv \underline{x}, \tag{3}$$

where, again, $\underline{v}_L < \overline{v}_L$ ensures that $\underline{x} < \overline{x}$.

Next, we can verify that T proposes $x^* = \overline{x}$ if $m_L^* = \overline{m}_L$ and $x^* = \underline{x}$ if $m_L^* = \underline{m}_L$, given its beliefs defined above. First, when $m_L^* = \overline{m}_L$, T proposes $x^* = \overline{x}$ rather than some $x' \geq \overline{x}$, since it will win L's acceptance but leave T strictly worse off than $\overline{x}$. It also will not propose some $x' < \overline{x}$, provoking rejection, as long as

$$1 - \overline{x} \geq \frac{m_T}{\overline{m}_L + m_T} - c_T, \tag{4}$$

which is guaranteed to be true since $c_L, c_T > 0$. Substituting $\underline{x}$ and $\underline{m}_L$ where appropriate also ensures that T will propose $x^* = \underline{x}$ following $m_L^* = \underline{m}_L$.

Finally, we verify that each player-type of L's prescribed action is incentive-compatible. First, the resolute type must choose $m_L^* = \overline{m}_L$ rather than deviate to $\underline{m}_L$, which convinces T that is is irresolute. If T believes it to be irresolute, then it offers terms that only the irresolute type will accept, forcing the irresolute type to reject in its return to optimal play. Therefore, the resolute L reveals its type via high mobilization when

$$-\overline{m}_L + \overline{v}_L\overline{x} \geq -\underline{m}_L + \frac{\underline{m}_L}{\underline{m}_L + m_T}\overline{v}_L - c_L \tag{5}$$

which is true as long as war is costly, $\underline{v}_L < \overline{v}_L$, and $m_T < \underline{v}_L$ (where the latter ensures that mobilization levels are positive). Finally, the irresolute type must choose $m_L^* = \underline{m}_L$ rather than deviate to $\overline{m}_L$, which convinces T that is is resolute. Since it is sure to accept any proposal that the resolute type does, its payoff to mimicking the resolute type involves accepting a relatively favorable proposal, meaning that it chooses low mobilization when $-\underline{m}_L + \underline{v}_L\underline{x} \geq -\overline{m}_L + \underline{v}_L\overline{x}$, which is true when

$$m_T \geq \frac{c_L^2\left(\sqrt{\overline{v}_L} + \sqrt{\underline{v}_L}\right)^2}{\overline{v}_L\left(\sqrt{\overline{v}_L} - \sqrt{\underline{v}_L}\right)^2} \equiv \hat{m}_T \tag{6}$$

Therefore, the proposed separating equilibrium exists when $m_T \geq \hat{m}_T$.

The semi-separating equilibrium occurs when $m_T < \hat{m}_T$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: with probability h, choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \underline{x}_h$; with probability $1-h$, $\underline{v}_L$ choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. T: believe $\phi' = \phi(h)$ if $m_L^* = \overline{m}_L$, then propose $x^* = \overline{x}$ with probability r and $x^* = \underline{x}_h$ with probability $1-r$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$, and propose $x^* = \underline{x}$. Again, there are no out-of-equilibrium beliefs.

Mobilization levels $\underline{m}_L$ and $\overline{m}_L$ are as defined above, and the resolute type's disincentive to deviate to $\underline{m}_L$ remains the same, since the target would again believe L to be irresolute and make a proposal that the resolute type is sure to reject. Likewise, T's optimal response to observing $\underline{m}_L$ is identical to that derived above. What remains is to establish the range of proposals $x \geq \underline{x}_h$ that $\underline{v}_L$ accepts after bluffing, T's posterior belief $\phi' = \phi(h)$ in the event of observing high mobilization, $\underline{v}_L$'s probability of choosing $\overline{m}_L$ that renders T indifferent between risking war and buying off both types, and T's probability of risking war that renders $\underline{v}_L$ indifferent between bluffing and honest revelation.

First, after choosing high mobilization, the irresolute L has a new expected value for war, which determines the proposals it accepts in equilibrium. It accepts proposals that satisfy

$$x\underline{v}_L \geq \frac{\overline{m}_L}{\overline{m}_L + m_T}\underline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{\sqrt{m_T}}{\sqrt{\overline{v}_L}} - \frac{c_L}{\underline{v}_L} \equiv \underline{x}_h. \tag{7}$$

Since types differ only in their valuation for war, the irresolute L is sure to accept both this proposal and any more generous one, i.e. $x \geq \overline{x}$, that the resolute type accepts. However, the resolute type will not accept $\underline{x}_h$, since its expected value for war is higher. This creates a risk-return tradeoff for T if it observes high mobilization: it can propose $x^* = \underline{x}_h$, which $\underline{v}_L$ accepts and $\overline{v}_L$ rejects, or $x^* = \overline{x}$, which both accept. As in similar models, it is straightforward to show that, as long as one type accepts a proposal, the costliness of war ensures that T never makes a proposal that both reject.

Second, let $\underline{v}_L$ choose $m^* = \overline{m}_L$ with probability h. Then, by Bayes' Rule, T's posterior belief that L is resolute, given $m^* = \overline{m}_L$, is $\Pr\left(\overline{v}_L|\overline{m}_L\right) = \phi / \left(\phi + h(1-\phi)\right)$. Third, $\underline{v}_L$ chooses the probability of bluffing, h, to render the target indifferent over proposing $\overline{x}$ and $\underline{x}_h$ after observing high mobilization. Formally, h solves

$$\frac{\phi}{\phi + h(1-\phi)}\left(\frac{m_T}{\overline{m}_L + m_T} - c_T\right) + \left(1 - \frac{\phi}{\phi + h(1-\phi)}\right)\left(1 - \underline{x}_h\right) = 1 - \overline{x},$$

where $\Pr\left(\overline{v}_L|\overline{m}_L\right)$ is as defined above. This yields its equilibrium probability of choosing high mobilization, h^*, where

$$h^* = \frac{\phi\left(c_L + c_T\overline{v}_L\right)\underline{v}_L}{(1-\phi)c_L\left(\overline{v}_L - \underline{v}_L\right)}.$$

Note that h^* takes on plausible values, $0 \leq h^* \leq 1$, when types are well-defined, or when $\underline{v}_L \leq \left(c_L\overline{v}_L\left(1-\phi\right)\right) / \left(c_L + \phi c_l \overline{v}_L\right)$. When types are well-defined, T finds it profitable to potentially separate them with its proposal—a precondition for a genuine risk of war—so I maintain this assumption throughout the analysis.

Finally, rendered indifferent over its available proposals, T chooses a probability of risking war, r, that renders $\underline{v}_L$ indifferent over bluffing and honestly revealing its type. Thus, r must solve $-\underline{m}_L + \underline{x}\left(\underline{v}_L\right) = \overline{m}_L + r\left(\underline{x}_h\underline{v}_L\right) + (1-r)\left(\overline{x}\underline{v}_L\right)$, yielding the equilibrium probability with which T makes the risky proposal,

$$r^* = 1 + \frac{\sqrt{m_T}\sqrt{\overline{v}_L}}{c_L}\left(1 - \frac{2}{1 + \sqrt{\underline{v}_L}/\sqrt{\overline{v}_L}}\right).$$

Algebra shows that T's mixing probability takes on plausible values $0 < r^* \leq 1$ when $m_T < \hat{m}_T$, as defined above, which establishes $\hat{m}_T$ as the cutpoint dividing the equilibrium space between pure separating and semi-separating equilibria.

It remains to show that pooling equilibria in which both player-types choose the same action do not exist. First, the resolute L's refusal to pool with the irresolute type, established by

Inequality (5), is strictly true given $\underline{v}_L < \overline{v}_L$ and any out-of-equilibrium beliefs T might hold, since L is sure to receive its reservation value after high mobilization regardless of T's strategy. Therefore, there can exist no equilibrium in which both types pool on low mobilization. Second, the irresolute type is unwilling to pool on high mobilization, since by assumption T's priors are sufficiently optimistic that it will risk war—that is, propose $\underline{v}_L$'s reservation value, which $\overline{v}_L$ goes on to reject—in a pooling equilibrium. For T to risk war, it must be the case that

$$\phi\left(\frac{m_T}{\overline{m}_L + m_T} - c_T\right) + (1-\phi)(1-\underline{x}_h) > 1 - \overline{x} \Leftrightarrow \phi < \frac{c_L\left(\overline{v}_L - \underline{v}_L\right)}{\overline{v}_L\left(c_L + c_T\underline{v}_L\right)} \equiv \phi_w.$$

To tolerate pooling under these conditions, the irresolute type would require $-\overline{m}_L + \underline{x}_h\underline{v}_L \geq -\underline{m}_L + \underline{x}\left(\underline{v}_L\right)$, which cannot be true as long as war is costly and $\underline{v}_L < \overline{v}_L$. Therefore, in the two-player case there are two equilibria: (1) a fully separating equilibrium when $m_T \geq \hat{m}_T$, and (2) a semi-separating equilibrium when $m_T < \hat{m}_T$. □

Proof of Proposition 2. I first prove the existence of separating and semi-separating equilibria, then show that pure pooling equilibria, on both high and low mobilizations, do not exist.

The separating equilibrium occurs when $m_T \geq \hat{m}_T$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. T: believe $\phi' = 1$ if $m_L^* = \overline{m}_L$ and propose $x^* = \overline{x}$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$ and propose $x^* = \underline{x}$. Since each mobilization level is chosen, there are no out-of-equilibrium beliefs. P: cooperate for all m_L.

To prove the existence of this equilibrium, begin by deriving each type of L's optimal mobilization level given its type and P's strategy. Since P cooperates for any mobilization choice, each type of L solves its maximization problem in the expectation of P's support. Thus, the resolute L chooses $\overline{m}_L$ such that it satisfies

$$\max_{m_L}\{-m_L + \frac{m_L + m_P}{m_L + m_P + m_T}\overline{v}_L - c_L\}.$$

The optimum satisfies the first-order condition, $m_T\overline{v}_L/(m_P + m_T + m_L)^2 - 1 = 0$, yielding

$$\overline{m}_L = \sqrt{m_T}\sqrt{\overline{v}_L} - m_T - m_P. \tag{8}$$

Substituting $\overline{m}_L$ into the second partial with respect to m_L yields $-2/\left(\sqrt{m_T}\sqrt{\overline{v}_L}\right) < 0$, ensuring that $\overline{m}_L$ is a maximum. Next, the irresolute L chooses $\underline{m}_L$ such that it satisfies

$$\max_{m_L}\{-m_L + \frac{m_L + m_P}{m_L + m_P + m_T}\underline{v}_L - c_L\}.$$

The optimum satisfies the first-order condition, $m_T\underline{v}_L/(m_P + m_T + m_L)^2 - 1 = 0$, yielding

$$\underline{m}_L = \sqrt{m_T}\sqrt{\underline{v}_L} - m_T - m_P. \tag{9}$$

Substituting $\underline{m}_L$ into the second partial with respect to m_L yields $-2/\left(\sqrt{m_T}\sqrt{\underline{v}_L}\right) < 0$, ensuring that $\underline{m}_L$ is a maximum.

Next, P cooperates for both low and high mobilization when $c_P < c_P^l$. To verify this, note that P cooperates after $\overline{m}_L$ when

$$\frac{\overline{m}_L + m_P}{\overline{m}_L + m_P + m_T}v_P - c_P\overline{m}_L > \frac{\overline{m}_L}{\overline{m}_L + m_T}v_P,$$

or when

$$c_P < \frac{v_P}{\overline{m}_L}\left(\frac{\overline{m}_L + m_P}{\overline{m}_L + m_P + m_T} - \frac{\overline{m}_L}{\overline{m}_L + m_T}\right) \equiv c_P^h. \tag{10}$$

Next, P cooperates after $\underline{m}_L$ when

$$\frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T} v_P - c_P \underline{m}_L > \frac{\underline{m}_L}{\underline{m}_L + m_T} v_P,$$

or when

$$c_P < \frac{v_P}{\underline{m}_L}\left(\frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T} - \frac{\underline{m}_L}{\underline{m}_L + m_T}\right) \equiv c_P^l. \tag{11}$$

Since $\underline{m}_L < \overline{m}_L$, as shown in Equations (8) and (9), it follows that $c_P^l < c_P^h$. Therefore, the lowest constraint binds, and P cooperates for both mobilization levels when $c_P < c_P^l$.

Next, consider what proposals L accepts after choosing m_L. The resolute type accepts iff x satisfies

$$x\overline{v}_L \geq \frac{\overline{m}_L + m_P}{\overline{m}_L + m_P + m_T}\overline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{c_L + \sqrt{m_T}\sqrt{\overline{v}_L}}{\overline{v}_L} \equiv \overline{x},$$

and the irresolute type accepts iff x satisfies

$$x\underline{v}_L \geq \frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T}\underline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{c_L + \sqrt{m_T}\sqrt{\underline{v}_L}}{\underline{v}_L} \equiv \underline{x}. \tag{12}$$

Note that, since $\underline{v}_L < \overline{v}_L$, it follows that $\underline{x} < \overline{x}$.

The next step is to verify that T proposes $x^* = \overline{x}$ if $m_L^* = \overline{m}_L$ and $x^* = \underline{x}$ if $m_L^* = \underline{m}_L$, given its beliefs in the separating equilibrium. First, when $m^* = \overline{m}_L$, T proposes $x^* = \overline{x}$ rather than some $x' > \overline{x}$, since the latter will win L's acceptance but leave T strictly worse off than $\overline{x}$. It also will not propose some $x' < \overline{x}$, provoking rejection, because

$$1 - \overline{x} \geq \frac{m_T}{\overline{m}_L + m_P + m_T} - c_T$$

is guaranteed to be true as long as $c_L, c_T > 0$. By the same logic, T will also make no proposal greater than $\underline{x}$ if $m_L^* = \underline{m}_L$, and it will propose no less when

$$1 - \underline{x} \geq \frac{m_T}{\underline{m}_L + m_P + m_T} - c_T, \tag{13}$$

which again is guaranteed to be true since $c_L, c_T > 0$.

Finally, we verify that each type of L's prescribed action is incentive-compatible. The resolute type must choose $m_L^* = \overline{m}_L$ rather than deviate to $\underline{m}_L$, which convinces T that it is irresolute. If T believes L to be irresolute, then it offers terms that only the irresolute type will accept, forcing $\overline{v}_L$ to reject in its return to optimal play. Therefore, $\overline{v}_L$ reveals its type via high mobilization when

$$-\overline{m}_L + \overline{x}\left(\overline{v}_L\right) \geq -\underline{m}_L + \frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T}\overline{v}_L - c_L,$$

which is guaranteed to be true since $\underline{v}_L < \overline{v}_L$ and $c_L > 0$. The irresolute type must choose $m_L^* = \underline{m}_L$ rather than deviate to $\overline{m}_L$, which convinces T that it is resolute. Since $\underline{v}_L$ is sure to accept any proposal that the resolute type accepts, its payoff to mimicking $\overline{v}_L$ involves accepting a relatively favorable proposal, meaning that it chooses low mobilization when $-\underline{m}_L + \underline{x}\left(\underline{v}_L\right) \geq -\overline{m}_L + \overline{x}\left(\underline{v}_L\right)$, which is true when

$$m_T \geq \frac{c_L^2\left(\sqrt{\overline{v}_L} + \sqrt{\underline{v}_L}\right)^2}{\overline{v}_L\left(\sqrt{\overline{v}_L} - \sqrt{\underline{v}_L}\right)^2} \equiv \hat{m}_T.$$

Note that this constraint is the same as that supporting the separating equilibrium in Proposition 1, defined in Inequality (6), because P's presence in the military balance is "canceled out" by L's adjustment of its mobilization decision. Therefore, the separating equilibrium exists for the conditions stated above.

The semi-separating equilibrium occurs when $m_T < \hat{m}_T$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: with probability h, choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \underline{x}_h$; with probability $1-h$, $\underline{v}_L$ choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. P: cooperate for all m_L. T: believe $\phi' = \phi(h)$ if $m_L^* = \overline{m}_L$, then propose $x^* = \overline{x}$ with probability r and $x^* = \underline{x}_h$ with probability $1-r$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$, and propose $x^* = \underline{x}$. As before, there are no out-of-equilibrium beliefs.

Mobilization levels $\underline{m}_L$ and $\overline{m}_L$ are as defined above, and the resolute type's disincentive to deviate to $\underline{m}_L$ remains the same, since the target would again believe L to be irresolute and make a proposal that the resolute type is sure to reject. Likewise, T's optimal response to observing $\underline{m}_L$ is identical to that derived above. What remains is to establish the range of proposals $x \geq \underline{x}_h$ that $\underline{v}_L$ accepts after bluffing, T's posterior belief $\phi' = \phi(h)$ in the event of observing high mobilization, $\underline{v}_L$'s probability of choosing $\overline{m}_L$ that renders T indifferent between risking war and buying off both types, and T's probability of risking war that renders $\underline{v}_L$ indifferent between bluffing and honest revelation.

First, after choosing high mobilization, the irresolute L has a new expected value for war, which determines the proposals it accepts in equilibrium. It accepts proposals that satisfy

$$x\underline{v}_L \geq \frac{\overline{m}_L + m_P}{\overline{m}_L + m_P + m_T}\underline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{\sqrt{m_T}}{\sqrt{\overline{v}_L}} - \frac{c_L}{\underline{v}_L} \equiv \underline{x}_h.$$

Since types differ only in their valuation for war, the irresolute L is sure to accept both this proposal and any more generous one, i.e. $x \geq \overline{x}$, that the resolute type accepts. However, the resolute type will not accept $\underline{x}_h$, since its expected value for war is higher. This creates a risk-return tradeoff for T if it observes high mobilization: it can propose $x^* = \underline{x}_h$, which $\underline{v}_L$ accepts and $\overline{v}_L$ rejects, or $x^* = \overline{x}$, which both accept. As in similar models, it is straightforward to show that, as long as one type accepts a proposal, the costliness of war ensures that T never makes a proposal that both reject.

Second, let $\underline{v}_L$ choose $m^* = \overline{m}_L$ with probability h. Then, by Bayes' Rule, T's posterior belief that L is resolute, given $m^* = \overline{m}_L$, is $\Pr\left(\overline{v}_L|\overline{m}_L\right) = \phi/\left(\phi + h(1-\phi)\right)$. Third, $\underline{v}_L$ chooses the probability of bluffing, h, to render the target indifferent over proposing $\overline{x}$ and $\underline{x}_h$ after observing high mobilization. Formally, h solves

$$\frac{\phi}{\phi+h(1-\phi)}\left(\frac{m_T}{\overline{m}_L+m_P+m_T}-c_T\right)+\left(1-\frac{\phi}{\phi+h(1-\phi)}\right)\left(1-\underline{x}_h\right)=1-\overline{x},$$

where $\Pr\left(\overline{v}_L|\overline{m}_L\right)$ is as defined above. This yields its equilibrium probability of choosing high mobilization, h^*, where

$$h^*=\frac{\phi\left(c_L+c_T\overline{v}_L\right)\underline{v}_L}{(1-\phi)c_L\left(\overline{v}_L-\underline{v}_L\right)}.$$

Note that h^* takes on plausible values, $0\le h^*\le 1$, when types are well-defined, or when $\underline{v}_L\le\left(c_L\overline{v}_L\left(1-\phi\right)\right)/\left(c_L+\phi c_l\overline{v}_L\right)$. When types are well-defined, T finds it profitable to potentially separate them with its proposal—a precondition for a genuine risk of war—so I maintain this assumption throughout the analysis.

Finally, rendered indifferent over its available proposals, T chooses a probability of risking war, r, that renders $\underline{v}_L$ indifferent over bluffing and honestly revealing its type. Thus, r must solve $-\underline{m}_L+\underline{x}\left(\underline{v}_L\right)=\overline{m}_L+r\left(\underline{x}_h\underline{v}_L\right)+(1-r)\left(\overline{x}\underline{v}_L\right)$, yielding the equilibrium probability with which T makes the risky proposal,

$$r^*=1+\frac{\sqrt{m_T}\sqrt{\overline{v}_L}}{c_L}\left(1-\frac{2}{1+\sqrt{\underline{v}_L}/\sqrt{\overline{v}_L}}\right).$$

Algebra shows that T's mixing probability takes on plausible values $0<r^*\le 1$ when $m_T<\hat{m}_T$, as defined above, which establishes $\hat{m}_T$ as the cutpoint dividing the equilibrium space between pure separating and semi-separating equilibria.

It remains to show that pooling equilibria in which both player-types choose the same action do not exist. First, the resolute L's refusal to pool with the irresolute type, established above, is strictly true given $\underline{v}_L<\overline{v}_L$ and any out-of-equilibrium beliefs T might hold, since L is sure to receive its reservation value after high mobilization regardless of T's strategy. Therefore, there can exist no equilibrium in which both types pool on low mobilization (or in which the resolute L probabilistically chooses low mobilization). Second, the irresolute type is unwilling to pool on high mobilization, since by construction T's priors are sufficiently optimistic that it will risk war—that is, propose $\underline{v}_L$'s reservation value, which $\overline{v}_L$ goes on to reject—in a pooling equilibrium. For T to risk war, it must be the case that

$$\phi\left(\frac{m_T}{\overline{m}_L+m_P+m_T}-c_T\right)+(1-\phi)(1-\underline{x}_h)>1-\overline{x}\Leftrightarrow\phi<\frac{c_L\left(\overline{v}_L-\underline{v}_L\right)}{\overline{v}_L\left(c_L+c_T\underline{v}_L\right)}\equiv\phi_w.$$

To tolerate pooling under these conditions, the irresolute type would require $-\overline{m}_L+\underline{x}_h\underline{v}_L\ge-\underline{m}_L+\underline{x}\left(\underline{v}_L\right)$, which cannot be true as long as war is costly and $\underline{v}_L<\overline{v}_L$. Therefore, in the three-player game with a committed partner, there are two equilibria: (1) a fully separating equilibrium when $m_T\ge\hat{m}_T$, and (2) a semi-separating equilibrium when $m_T<\hat{m}_T$. Since L's mobilization decisions take P's contribution into account for both low and high mobilization, the equilibria and the threshold dividing them are the same as in the two-player game. □

Proof of Proposition 3. I first prove the existence of separating, semi-separating, and pooling equilibria, then show that equilibria in which players pool on high mobilization and in which the resolute L probabilistically chooses low mobilization do not exist.

The separating equilibrium exists when $c_P^l \leq c_P < c_P^h$ and $m_T \geq \max\{m_T^\dagger, \tilde{m}_T\}$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. P: cooperate iff $m_L^* = \underline{m}_L$. T: believe $\phi' = 1$ if $m_L^* = \overline{m}_L$ and propose $x^* = \overline{x}$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$ and propose $x^* = \underline{x}$. Since each possible mobilization level is chosen, there are no out-of-equilibrium beliefs.

To prove the existence of this equilibrium, begin by deriving each type of L's optimal mobilization level given its type and P's strategy. Since P cooperates only if $m_L^* = \underline{m}_L$, the resolute L's optimal mobilization, $\overline{m}_L$, is the same as given above in Equation (1). The irresolute L expects P's support, so its optimum, $\underline{m}_L$, is the same as given in Equation (9).

Given mobilization levels, we can now derive the conditions that support P's strategy of cooperating if $m_L^* = \underline{m}_L$ and defecting if $m_L^* = \overline{m}_L$. Formally, this requires that both

$$\frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T} v_P - c_P \underline{m}_L \geq \frac{\underline{m}_L}{\underline{m}_L + m_T} v_P$$

and

$$\frac{\overline{m}_L}{\overline{m}_L + m_T} v_P > \frac{\overline{m}_L + m_P}{\overline{m}_L + m_P + m_T} v_P - c_P \overline{m}_L$$

be simultaneously true. This system of inequalities is true when $c_P^l \leq c_P < c_P^h$, where c_P^h and c_P^l are as defined in Inequalities (10) and (11), respectively.

Next, consider what proposals L accepts after choosing m_L. The resolute type accepts any $x \geq \overline{x}$, as defined by Inequality (2), since its payoffs are identical to the separating equilibrium in the two-player game. The irresolute type accepts any $x \geq \underline{x}$, as defined by Inequality (12), since its payoffs are identical to the separating equilibrium in the three-player game with a committed partner. Further, $\underline{x}$ is also the same proposal defined by Inequality (3), since L's mobilization decision "cancels out" the effect of m_P on the proposal. Thus, as before, $\underline{x} < \overline{x}$.

Next, we can verify that T proposes $x^* = \overline{x}$ if $m_L^* = \overline{m}_L$ and $x^* = \underline{x}$ if $m_L^* = \underline{m}_L$, given its beliefs defined above. First, when $m_L^* = \overline{m}_L$, T proposes $x^* = \overline{x}$ rather than some $x' \geq \overline{x}$, since it will win L's acceptance but leave T strictly worse off than $\overline{x}$. It also will not propose some $x' < \overline{x}$, provoking rejection, as long as $c_L, c_T > 0$, as defined in Inequality (4). By the same logic, T will also make no proposal greater than $\underline{x}$ if $m_L^* = \underline{m}_L$, nor will it propose any less, since $c_L, c_T > 0$ as shown for the identical decision in Inequality (13).

Finally, we verify that each player-type of L's prescribed action is incentive-compatible. First, the resolute type must choose $m_L^* = \overline{m}_L$ rather than deviate to $\underline{m}_L$, which convinces T that is is irresolute and wins P's cooperation. If T believes it to be irresolute, then it offers terms that only the irresolute type will accept, forcing the irresolute type to reject in its return to optimal play. Therefore, the resolute L reveals its type via high mobilization when

$$-\overline{m}_L + \overline{x}\left(\overline{v}_L\right) \geq -\underline{m}_L + \frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T} \overline{v}_L - c_L, \tag{14}$$

which is true when

$$m_T \geq \frac{m_P^2 \underline{v}_L}{\left(\sqrt{\overline{v}_L} - \sqrt{\underline{v}_L}\right)^4} \equiv m_T^{\dagger}. \tag{15}$$

Finally, the irresolute type must choose $m_L^* = \underline{m}_L$ rather than deviate to $\overline{m}_L$, which convinces T that it is resolute and ensures P's cooperation. Since it is sure to accept any proposal that the resolute type does, its payoff to mimicking the resolute type involves accepting a relatively favorable proposal, meaning that it chooses low mobilization when $-\underline{m}_L + \underline{x}\left(\underline{v}_L\right) \geq -\overline{m}_L + \overline{x}\left(\underline{v}_L\right)$, which is true when

$$m_T \geq \frac{\left((-c_L + m_P)\,\overline{v}_L + c_L \underline{v}_L\right)^2}{\overline{v}_L\left(\sqrt{\overline{v}_L} - \sqrt{\underline{v}_L}\right)^4} \equiv \tilde{m}_T. \tag{16}$$

Therefore, the separating equilibrium exists when $c_P^l \leq c_P < c_P^h$ and $m_T \geq \max\{\tilde{m}_T, m_T^{\dagger}\}$.

The semi-separating equilibrium exists when $c_P^l \leq c_P < c_P^h$ and $m_T^{\dagger} \leq m_T < \tilde{m}_T$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \overline{x}$. $\underline{v}_L$: with probability h, choose $m_L^* = \overline{m}_L$ and accept iff $x \geq \underline{x}_h$; with probability $1-h$, $\underline{v}_L$ choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$. P: cooperate iff $m_L^* = \underline{m}_L$. T: believe $\phi' = \phi(h)$ if $m_L^* = \overline{m}_L$, then propose $x^* = \overline{x}$ with probability r and $x^* = \underline{x}_h$ with probability $1-r$; believe $\phi' = 0$ if $m_L^* = \underline{m}_L$, and propose $x^* = \underline{x}$. As before, there are no out-of-equilibrium beliefs.

Mobilization levels $\underline{m}_L$ and $\overline{m}_L$ are as defined in the separating equilibrium, and the condition supporting the resolute type's disincentive to deviate to $\underline{m}_L$ remains the same, $m_T \geq m_T^{\dagger}$, since the target would again believe L to be irresolute and make a proposal that the resolute type is sure to reject. Likewise, T's optimal response to observing $m_L^* = \underline{m}_L$ is identical to that derived for the separating equilibrium immediately above, as is the range of proposals the irresolute type accepts if it chooses high mobilization, i.e. $\underline{x}_h$ from Inequality (7), and T's choice between buying off both types or risking war. What remains is to establish T's posterior belief $\phi' = \phi(h)$ in the event of high mobilization, $\underline{v}_L$'s probability of choosing $\overline{m}_L$ that renders T indifferent between risking war and buying off both types, and T's probability of risking war that renders $\underline{v}_L$ indifferent between bluffing and honest revelation.

The first step is to derive the probability with which $\underline{v}_L$ chooses $m_L^* = \overline{m}_L$, or h. By Bayes' Rule, T's posterior belief that L is resolute, given $m^* = \overline{m}_L$, is $\Pr\left(\overline{v}_L | \overline{m}_L\right) = \phi / \left(\phi + h(1-\phi)\right)$. Third, $\underline{v}_L$ chooses the probability of bluffing, h, to render the target indifferent over proposing $\overline{x}$ and $\underline{x}_h$ after observing high mobilization. Formally, h solves

$$\frac{\phi}{\phi + h(1-\phi)}\left(\frac{m_T}{\overline{m}_L + m_T} - c_T\right) + \left(1 - \frac{\phi}{\phi + h(1-\phi)}\right)\left(1 - \underline{x}_h\right) = 1 - \overline{x},$$

where $\Pr\left(\overline{v}_L | \overline{m}_L\right)$ is as defined above. This yields its equilibrium probability of choosing high mobilization, h^*, where

$$h^* = \frac{\phi\left(c_L + c_T \overline{v}_L\right)\underline{v}_L}{(1-\phi)c_L\left(\overline{v}_L - \underline{v}_L\right)}.$$

Note that h^* takes on plausible values, $0 \leq h^* \leq 1$, when types are well-defined, or when $\underline{v}_L \leq \left(c_L \overline{v}_L\left(1-\phi\right)\right) / \left(c_L + \phi c_l \overline{v}_L\right)$ as established above.

Finally, rendered indifferent over its available proposals, T chooses a probability of risking war, r, that renders $\underline{v}_L$ indifferent over bluffing and honestly revealing its type. Thus, r must solve $-\underline{m}_L + \underline{x}\left(\underline{v}_L\right) = \overline{m}_L + r\left(\underline{x}_h \underline{v}_L\right) + (1-r)\left(\overline{x}\underline{v}_L\right)$, yielding the equilibrium probability with which T makes the risky proposal,

$$r^* = \frac{-m_P \overline{v}_L - \sqrt{m_T}\sqrt{\overline{v}_L}\left(\sqrt{\overline{v}_L} - \sqrt{\underline{v}_L}\right)^2 + c_L\left(\overline{v}_L - \underline{v}_L\right)}{c_L\left(\overline{v}_L - \underline{v}_L\right)}$$

Algebra shows that T's mixing probability takes on plausible values $m_T < \tilde{m}_T$. Therefore, the semi-separating equilibrium exists when $c_P^l \leq c_P < c_P^h$ and $m_T^\dagger \leq m_T < \tilde{m}_T$.

The pooling equilibrium in which both types choose low mobilization occurs when $m_T < \min\{m_T^\dagger, \hat{m}_T\}$. Strategies and beliefs are as follows. $\overline{v}_L$: choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \overline{x}_l$; if $m_L^* = \overline{m}_L$, accept iff $x \geq \overline{x}$. $\underline{v}_L$: choose $m_L^* = \underline{m}_L$ and accept iff $x \geq \underline{x}$; if $m_L^* = \overline{m}_L$, accept iff $x \geq \underline{x}_h$. P: cooperate iff $m_L^* = \underline{m}_L$. T: believe (in equilibrium) $\phi' = \phi$ if $m_L^* = \underline{m}_L$, and propose $x^* = \underline{x}$; believe (out of equilibrium) $\phi' = 0$ if $m_L^* = \overline{m}_L$, then propose $x^* = \underline{x}$.

Mobilizations $\underline{m}_L$ and $\overline{m}_L$ are as defined above, and the proposals each type accepts—$x \geq \underline{x}$ for $\underline{v}_L$ if $m_L^* = \underline{m}_L$, $x \geq \underline{x}_h$ for $\underline{v}_L$ if $m_L^* = \overline{m}_L$, and $x \geq \overline{x}$ for $\overline{v}_L$ if $m_L^* = \overline{m}_L$—are as defined in the separating and semi separating equilibria. It remains to determine the proposals that the resolute L accepts following $m_L^* = \underline{m}_L$, to verify that T risks war following $m_L^* = \underline{m}_L$, and identifying conditions under which each type of L takes its prescribed actions.

First, after choosing low mobilization, the resolute L accepts proposals that satisfy

$$x\overline{v}_L \geq \frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T}\overline{v}_L - c_L \Leftrightarrow x \geq 1 - \frac{c_L}{\overline{v}_L} - \frac{\sqrt{m_T}}{\sqrt{\overline{v}_L}} \equiv \overline{x}_l. \tag{17}$$

Since types differ only in their valuation of the stakes, the irresolute L is sure to accept both this proposal and any more generous one, i.e. $x \geq \overline{x}_l$ that the resolute type accepts. However, the resolute type will not accept $\underline{x}$, since its expected value for war is higher. This creates a risk-return tradeoff for T if it observes low mobilization: it can propose $x^* = \underline{x}$, which $\underline{v}_L$ accepts and $\overline{v}_L$ rejects, or $x^* = \overline{x}_l$, which both accept. As in similar models, it is straightforward to show that, as long as one type accepts a proposal, the costliness of war ensures that T never makes a proposal that both types are sure to reject.

Next, we verify that on the equilibrium path T proposes $x^* = \underline{x}$ when $\phi < \phi_w$ after $m_L^* = \underline{m}_L$. As before, T will never concede any more than necessary to induce a type's acceptance, nor will it make a proposal that neither type accepts. Therefore, it proposes $x^* = \underline{x}$, generating a risk of war, when

$$\phi\left(\frac{m_T}{\underline{m}_L + m_P + m_T} - c_T\left(\underline{m}_L + m_T\right)\right) + \left(1-\phi\right)\left(1-\underline{x}\right) > 1 - \overline{x}_l \Leftrightarrow \phi < \frac{c_L\left(\overline{v}_L - \underline{v}_L\right)}{\overline{v}_L\left(c_L + c_T\underline{v}_L\right)} \equiv \phi_w,$$

as stated above.

Finally, we show that each type of L's strategy is incentive-compatible. Since both types choose $m_L^* = \underline{m}_L$ in equilibrium, we must first define T's out-of-equilibrium beliefs in the event that it observes high mobilization. As stated above, I assign beliefs $\phi' = 0$ in the event of high mobilization, i.e. that T believes that only the *irresolute* type would deviate to high mobilization. While this seems counterintuitive, it is nonetheless the *only* belief that satisfies Cho and

Kreps' (1987) Intuitive Criterion when $\underline{v}_L$ has an incentive to bluff, or when $m_T < \tilde{m}_T$, for two reasons. First, the resolute type never does better by deviating to high mobilization, regardless of T's strategy, because it can always guarantee itself its reservation value should it deviate from low mobilization; thus, if it prefers receiving its reservation value under low mobilization to high, then it cannot profit by deviating to high mobilization for *any* of T's possible strategies. Second, however, the irresolute type *can* do better than its equilibrium payoff for *one* of T's possible strategies—that is, if T will propose the resolute type's reservation price—regardless of whether that strategy will be played off the equilibrium path (see also Fudenberg and Tirole 1991, pp. 447-451). Therefore, should T observe high mobilization, it believes that only the irresolute type would have attempted this deviation and as such proposes its reservation value, $\underline{x}_h$, which the irresolute type accepts but which the resolute type rejects.

It is worth noting that, since the irresolute type has no incentive to bluff about its resolve when $m_T \geq \tilde{m}_T$, any set of out-of-equilibrium beliefs and consistent strategies for T support the equilibrium. Thus, while the equilibrium is supported under a wider range of conditions when $m_T \geq \tilde{m}_T$, I characterize results based on out-of-equilbirium beliefs $\phi' = 0$, because $\underline{v}_L$ does weakly worse than the resolute type along any equilibrium path and can be considered the type with the most incentive to deviate even if the Intuitive Criterion does not strictly apply (Fudenberg and Tirole 1991, p. 447), as is the case when $m_T \geq \tilde{m}_T$.

Begin with the resolute type. When it chooses $m_L^* = \underline{m}_L$, T cannot update its beliefs and so proposes $x^* = \underline{x}$, which $\overline{v}_L$ rejects since $\underline{x} < \overline{x}_l$; should it deviate and choose high mobilization, T believes it to be irresolute and proposes terms, $\underline{x}_h$, that the resolute type is sure to reject (however, note that by rejecting, $\overline{v}_L$ gets in expectation exactly what it would if T believed L to be resolute and proposed $\overline{x}$). Therefore, the resolute L chooses low mobilization when

$$-\underline{m}_L + \frac{\underline{m}_L + m_P}{\underline{m}_L + m_P + m_T}\overline{v}_L - c_L \geq -\overline{m}_L + \frac{\overline{m}_L}{\overline{m}_L + m_T}\overline{v}_L - c_L,$$

or when $m_T \leq m_T^{\dagger}$, where $m_T^{\dagger}$ is as defined in Inequality (15). Next, consider the irresolute type. When it chooses $m_L^* = \underline{m}_L$, T proposes $x^* = \underline{x}$, which it accepts; should it deviate and choose high mobilization, T believes it to be irresolute, and it proposes $\underline{x}_h$, which $\underline{v}_L$ accepts. Therefore, the irresolute L chooses low mobilization when $-\underline{m}_L + \underline{x}\left(\underline{v}_L\right) \geq -\overline{m}_L + \underline{x}_h\left(\underline{v}_L\right)$, which is guaranteed to be true since $\underline{v}_L < \overline{v}_L$ and $c_L > 0$. However, note that the irresolute L *would* profit from a deviation if T's out-of-equilibrium beliefs were such that it proposed the resolute type's reservation value under high mobilization, since $-\overline{m}_L + \overline{x}\left(\underline{v}_L\right) > -\underline{m}_L + \underline{x}\left(\underline{v}_L\right)$ when $m_T < \tilde{m}_T$. Thus, since at least one of T's possible out-of-equilibrium strategies can be profitable for $\underline{v}_L$—where the same is not true for the resolute type—T's belief $\phi' = 0$ satisfies the Intuitive Criterion. Therefore, the pooling equilibrium in which both types of L choose $m_L^* = \underline{m}_L$ exists when $c_P^l \leq c_P < c_P^h$ and $m_T < m_T^{\dagger}$.

It remains to show that there exists no pooling equilibrium in which both types of L choose high mobilization and no semi-separating equilibrium in which the resolute L probabilistically chooses low mobilization. First, consider a potential pooling equilibrium on high mobilization. The irresolute L must be willing to accept its minmax payoff following $m^* = \overline{m}_L$ if T chooses to make a proposal that only the irresolute type accepts—which T does by construction—rather than revealing itself as irresolute and receiving its minmax payoff under a lower mobilization level. Formally, this requires $-\overline{m}_L + \underline{x}_h\left(\underline{v}_L\right) \geq -\underline{m}_L + \underline{x}\left(\underline{v}_L\right)$, which cannot be true as long as war is costly and $\underline{v}_L < \overline{v}_L$. Therefore, such a pooling equilibrium cannot exist. Second, consider a semi-separating equilibrium in which the resolute L probabilistically chooses low mobilization. This requires that the resolute type can be rendered indifferent over revealing itself as resolute with high mobilization and choosing low mobilization, after which T randomizes its offers. However, in any equilibrium, the resolute type does no better than its minmax value (given the ultimatum bargaining protocol), so it cannot be rendered indifferent across the two actions, since T's probability of risking war or not has no effect on $\overline{v}_L$'s payoffs. Thus, there can be no semi-separating equilibrium in which the resolute L probabilistically masks its type. Therefore, in the three-player case with a skittish partner there are three equilibria: (1) a fully separating equilibrium, (2) a semi-separating equilibrium in which the resolute type bluffs probabilistically, and (3) a pooling equilibrium in which both types of L choose low mobilization. □

References

Cho, In-Koo and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics* 102(2):179–221.

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory*. Cambridge: MIT Press.