Coalition Politics and War Termination: On the "Early" End of the First World War*

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Abstract

The theory of war termination accounts for fighting that ends when it solves the bargaining problem that prevented settlement in the first place. Yet some wars end before and others after that point. I propose a solution to this puzzle by adding a coalition partner to a two-player model of war termination and allowing power to shift both across and within sides. Continuing the war stabilizes the distribution of power with a coalition's enemy, but it also shifts relative power inside the coalition, whose members disagree over how to share the postwar pie. In equilibrium, coalition politics can lengthen or shorten war, depending on (a) whether fighting has solved the enemy's commitment problem, (b) whether power is shifting between coalition partners, and (c) the costs of intra-coalition discord. I illustrate the model's usefulness by explaining why World War I ended with 1918's armistice and not an Allied drive into Germany.

Word Count

We both know the paper's too long. Let's leave it at that, shall we?

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The theory of war termination accounts for fighting that ends when it solves the bargaining problem that prevented a negotiated settlement in the first place, resolving uncertainty about the relative attractiveness of war (e.g. Filson and Werner 2002) or stabilizing the distribution of power (e.g. Leventoğlu and Slantchev 2007). Wars end "on time" when their underlying bargaining problems have been solved, saving belligerents the costs of further fighting. Yet some wars end "late" (Goemans 2000, Maoz and Siverson 2008) and others end "early" (Beard 2019, Werner and Yuen 2005). The Paraguayan War, for example, ended more than a year after the Triple Alliance stabilized the distribution of power in South America.[1] Brazil, Argentina, and Uruguay had captured Ascunción, defeated Paraguay's main military force, stripped it of disputed territories, and set up a provisional government. The end of Paraguay's ambitions notwithstanding, the victors kept up the war for an additional year (Abente 1987, 64-65, Leuchars 2002). On the other hand, the November 1918 armistice that ended fighting in World War I left German territory unconquered, its armies intact, and its generals considering a resumption of hostilities from shorter interior lines if post-armistice negotiations went poorly. The Allies, unaware of the true scale of Germany's defeat (Stevenson 2005, 108), granted armistice despite worries that Germany might exploit the pause to resume its bid for hegemony. Why do belligerents end some wars well after solving the underlying bargaining problem while others end them well before?

Solving the puzzle requires a model in which belligerents can end wars early, late, or on time, but most work focuses either late or early terminations. Explanations for late terminations include political survival incentives (Arena 2015, Croco 2015, Goemans 2000), the costs of peacetime debt (Slantchev 2012), organizational biases (Gartner 1997), political/social psychology (Stanley 2009, Stanley and Sawyer 2009, Sticher 2021), and battlefield advantages (Beard 2019, Ch. 3), yet they're mostly silent on early terminations. Accounts of early terminations, on the other hand, focus on exhaustion or third-party imposition (Werner 1999, Werner and Yuen 2005), taking the decision to end the war out of belligerents' hands and leaving open the question of late terminations. Powell (2017) relaxes the two-player assumption in a model of third-party intervention and war duration, but players don't bargain, war may not be costly, and the third party has preferences only over which belligerent wins the war. The Paraguayan and First World Wars also point to relaxing the two-player assumption, but they highlight intra-coalition disagreements over how to divide the pie, which makes bargaining essential to the story. Brazil and Argentina each cared as much about ensuring that the other wouldn't emerge from the Paraguayan War advantaged as it did about defeating Paraguay. Likewise, the Entente in 1918 minded the effect of continued fighting on both Germany's ability to renege on an armistice and the United States' ability to dominate an eventual settlement. This implies a model of coalition politics and war duration with distributive conflicts both across and within warring sides.

I analyze a three-player model of war termination that reflects this tension. First, coalition partners agree on reducing the enemy's share of the pie but also wish to gain at each other's expense (see also Phillips and Wolford 2021). Second, relative power may shift between partners if fighting continues, allowing one partner to claim a larger share of the postwar pie. Belligerents may capture strategic territory, like Serbia seizing Macedonia

[1] This conflict is also known as the López War, after Paraguay's military dictator, and the War of the Triple Alliance, after the victors. I've chosen the name that reflects where its staggering number of victims died.

in the First Balkan War (Dakin 1962, 26-30; Glenny 2012, 243-248), or they may take on increasing shares of the military and financial burden over time, like the United States during World War I (see McCrae 2019) or Brazil in the Paraguayan War (Strauss 1978). This within-side bargaining problem influences how the coalition deals with the across-side bargaining problem, in which relative power will shift in an enemy's favor unless fighting prevents it (see Powell 2012, Wolford, Reiter and Carrubba 2011). Continued fighting may solve one commitment problem at the expense of creating another. Belligerents can choose to fight until one side is eliminated, which strengthens one coalition partner, or to end the war in a negotiated settlement, after which the coalition confronts a more powerful enemy. Finally, coalition politics determines which partner casts the decisive vote on war termination, where an open breach undermines the pursuit of other coalition goals. Unlike Powell's (2017) model, sides are fixed, but this allows for a richer analysis of the relationship between coalition politics, intra-war negotiations, and war termination.

The model accounts for wars that end early, late, on time while retaining the core assumptions of the theory of endogenous war termination. Players bargain over a continuously-divisible pie, and war is costly, yet the war entails two commitment problems. Just as a rising enemy can't promise not to leverage future strength if the war ends in a negotiated settlement, a rising coalition partner can't promise to demand a larger share of the pie from its partner if the war continues. In equilibrium, the timing of war termination depends on (a) whether fighting has already stabilized the across-side distribution of power, (b) whether power is shifting between coalition partners, and (c) the costs coalition members pay for disagreeing over war aims. When intra-coalition power is stable, war ends once fighting solves the enemy's commitment problem and lasts no longer. But when intra-coalition power is shifting, wars can end early or late. When the across-side bargaining problem has been solved, shifting intra-coalition power can extend the war if the costs of coalition discord are low. When power is still shifting across sides, shifting intra-coalition power can cause an *early* end when the costs of coalition discord are sufficiently high.[2] Therefore, coalition politics can hasten *or* delay war termination, accounting for late, on-time, the latter of which I illustrate by discussing the "early" end of World War I on the Western Front in 1918.

Coalitions and War Termination

Military coalitions have fought in 40% of interstate wars since 1816 (Sarkees and Wayman 2010), yet most models of endogenous war termination are dyadic (see, *inter alia*, Filson and Werner 2002, Langlois and Langlois 2009, Leventoğlu and Slantchev 2007, Powell 2004, Slantchev 2003, Thomas, Reed and Wolford 2016, Wolford, Reiter and Carrubba 2011). The two-player assumption is innocuous where coalitions are equal to the sums of their parts, but military coalitions aren't mere aggregations of military power (Wolford 2015, 41-45).[3] Coalition politics determines how partners share the costs and benefits of fighting (Riker

[2] In an appendix, I describe a separate mechanism that can extend or shorten the war, when the partner's rise implicates the across-side distribution of power but not the intra-coalition distribution of power.

[3] Wolford shows that, while coalitions and singleton states differ substantially in the rates at which their international crises escalate to war, most of the difference is accounted for by factors other than differences in size and military capabilities.

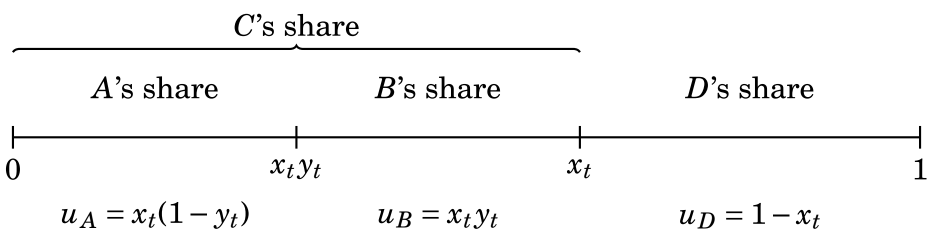
1962), shaping the escalation of international crises to war (Kreps 2011, Wolford 2015, Ch. 4), their expansion (*ibid.*, Ch. 5.), and their duration (Chiba and Johnson 2019), as well as battlefield performance (Zielinski and Grauer 2020), war outcomes (Gartner and Siverson 1996, Graham, Gartzke and Fariss 2017, Morey 2016, 2020), shares of the postwar pie (Starr 1972), and the durability of peace settlements (Phillips and Wolford 2021, Wolford 2017). It stands to reason that coalition politics also affects war duration, yet neither of the two prevailing stories linking coalition politics to war duration—collective action and coalition size—offers a consistent account of wars ending early, on time, and late.

Several arguments link collective action problems to an under-provision of war effort (see, e.g. Auerswald and Saideman 2013, Choi 2012, Olson and Zeckhauser 1966, Weitsman 2004, 2010). Bennett and Stam (1996, 423-424), for example, conjecture that collective action problems shorten coalition conflicts by undermining the ability to wage war to a victorious conclusion. Yet some coalition wars last mere weeks, like the 1991 Gulf War, and others, like the Korean War, last years; further, the shorter of the two was a coalition victory, the longer one a draw. Other work questions the systematic importance of wartime collective action problems, since they're often solved on the equilibrium path. First, coalition-builders often compensate their partners to offset the private costs of the war effort, which reduces free-riding (Henke 2017, 2019, Wolford 2015, Wolford and Ritter 2016). Second, partners may fight to preserve their reputations for honoring commitments (Leeds 2003*a*, Morrow 1994) or make the case for stronger ties with their partners (Gannon and Kent 2020), obtaining private benefits unrelated to the outcome of the war in return for contributing to the collective good. Finally, belligerents may disagree with their partners over how to divide the postwar pie (cf. Phillips and Wolford 2021) and fight to position themselves favorably for postwar negotiations. For example, collective action problems weren't a dominant issue for the Allies during First World War. From the First Battle of the Marne in 1914 through the Hundred Days offensive of 1918, the British, French, and Americans all had clear and competing private incentives—all of which depended on a dominant seat at the negotiating table—to contribute to the collective military effort (see Wolford 2019, Ch. 7).

A second set of claims relates larger numbers of interested parties to bargaining inefficiencies (see, e.g. Bas and Schub 2016, Chiba and Johnson 2019, Gallop 2017, Huth, Bennett and Gelpi 1992, Lake 2010/11). Blainey (1988, 197) and Vasquez (1993, 258–260), for example, contend that more participants, whether or not they fight as coalitions, imply longer interstate wars. Cunningham (2006) finds that civil wars with more parties last longer than civil wars with fewer parties, though Findley and Rudloff (2012) find that rebel group fragmentation during war can be associated with longer or shorter civil wars. Arguments about multiple parties follow a shared line of reasoning; the more parties that can veto peace by staying in the fight or the more parties whose uncertain interests must be judged in formulating war aims, the harder it is to find a compromise and the longer the war on average. Yet in many cases, like collective deterrence (Fang, Johnson and Leeds 2014, Leeds 2003*b*, Phillips and Wolford 2021, Wolford 2017), coercive bargaining (Wolford 2014, Wolford 2015, Ch. 4), and third-party interventions (Favretto 2009, Powell 2017, Yuen 2009), the strategic involvement of more actors can *reduce* inefficiency by solving information problems or smoothing out shifts in power. Finally, coalition politics are often defined by attempts to mitigate problems of size (Fordham and Poast 2016, Riker 1962, Wolford 2015, Ch. 5).

Neither collective action nor coalition size provides a consistent account of how coalition

Figure 1: A Two-Dimensional Peace Settlement



politics shapes war termination. Each perspective elides distributive competition inside military coalitions, which shapes the terms on which partners are willing to terminate the war. Collective action accounts necessarily deal with public goods—say, victory itself—as opposed to private goods like territory and influence, and coalition size accounts rarely offer an explicit model of *how* preferences are aggregated inside coalitions. My approach accounts for both elements of coalition war. First, coalition partners disagree over how to share the postwar pie, and bargaining between coalition partners need not be frictionless. The course of fighting may shift relative power between coalition members, as it did for the Triple Alliance during the Paraguayan War and the Allies during World War I, confronting them with not just across-side but also within-side bargaining frictions. Second, I model coalition politics as internal negotiations over both war aims and shares of the pie, which allows for an explicit link between the presence of coalition partners and the duration of war.

Model

Suppose that a coalition of two partners $C = \{A, B\}$ and an enemy D must choose whether to terminate or continue an ongoing war.[4] The war may end in a negotiated settlement in which no players are eliminated or a total war which eliminates the losing side. Whether the war ends or continues in the first period, surviving players negotiate in the shadow of war in the second period. If war ends in the first period, no player is eliminated and each receives a share of the unit pie, though D may grow stronger ahead of renegotiations in the second period. If war continues in the first period, one side eliminates the other and secures the pie for the remainder of the game. If the coalition eliminates D in the first period, its members must still negotiate their own shares of the pie, and B may have grown stronger relative to A over the course of the fighting. Terminating the war too soon may fail to solve D's commitment problem, but continuing the war may create a commitment problem for B, whose rise may allow it to demand a larger share of the postwar pie.

Figure 1 shows that settlements are defined by an across-side bargain that allocates

[4] Recent theories of war termination follow Wagner's (2000) advice to allow also for endogenous initiation (e.g. Filson and Werner 2002, Leventoğlu and Slantchev 2007, Powell 2004, Spaniel and Bils 2018, Wolford, Reiter and Carrubba 2011), but it's not clear that war initiation is necessary to solve the present puzzle. In 1914, neither the Entente nor Germany anticipated a long war on the Western Front (Debs 2020), much less one that would involve the United States in the final settlement as an emergent hegemon.

Figure 2: The first period in timeline form

- D proposes x_1
- A accepts or rejects
- If A accepts, B accepts or rejects
- War continues unless both accept
- A proposes y_1 if war ended, w_1 if war continued
- B accepts or rejects
- Intra-coalition war if B rejects
- Play continues at $t = 2$
  - If peace, repeat first stage
  - If war, A v. B, D v. A, or D v. B

$x_t \in [0,1]$ to the coalition and $1-x_t$ to D and a within-side bargain that allocates a share $y_t \in [0,1]$ of x_t to B and $1-y_t$ to A. Payoffs are linear in shares of the unit pie. Figure 2 shows that play begins at $t = 1$ with D proposing x_1 to the coalition. Coalition politics are defined by B's inability to commit to ending the war if continued fighting is sufficiently attractive, so A responds first; otherwise, B could dictate continued fighting, rendering coalition politics moot. If A rejects D's offer, the war continues to a decisive outcome that eliminates the losing side, C or D. If A accepts, B has an opportunity to accept or reject, where acceptance implements D's proposed across-side bargain, giving $1-x_1$ to D for the rest of the period, but rejection continues the war. If B overturns A by continuing the war, each partner's payoffs are reduced by a share $c \in (0,1)$, which represents the costs of coalition discord—i.e., any costs for a public breach paid by the coalition on issues unrelated to the distribution of the pie. A public breach can entail delay or friction in implementing war plans, counterproductive signals to friendly and enemy audiences, compromised deterrence of postwar threats, and simple bad blood. If the war is to continue, partners prefer not to disagree about it beforehand. This representation of coalition politics rules out defections (Choi 2012, Weisiger 2016), side-changing (Powell 2017, 232), and side payments (Henke 2019, Wolford 2015) by which one partner might prevent the other from continuing the war. But even if one partner forces the other to end the fighting, or even if the enemy were to split the coalition (Crawford 2011), the reason behind the desire to continue—shifting power—would be the same.

If the coalition accepts D's proposal, A and B then negotiate over their shares of the across-side bargain (x_1). A proposes a share $y_1 \in [0,1]$ to B, who accepts or rejects. Acceptance allocates $x_1(1-y_1)$ to A and $x_1 y_1$ to B. Rejection leads to an intra-coalition war in which one coalition partner eliminates the other for sole control of x_t at $t = 1$, until negotiations with D over the entire pie at $t = 2$. If the coalition rejects D's proposal and wins the resulting war, the coalition bargains over the entire unit pie, such that A offers a share

$w_1 \in [0,1]$ that B can then accept or reject. The latter means intra-coalition war, though the winner won't face the just-eliminated D at $t = 2$.

The first period can end in settlements with or without intra-coalition war, continued war that D wins, or continued war that C wins and after which A and B may or may not fight. Payoffs accrue, then play moves to $t = 2$, payoffs for which players discount at rate $\delta \in (0,1)$. If war ended in the first period, the second period is structurally identical. D proposes x_2, which the coalition accepts or rejects, after which partners bargain around y_2 or w_2 in the shadow of intra-coalition war. Any war in the second period, whether across or within sides, eliminates the loser.[5] If D defeated C at $t = 1$, it enjoys the entire pie. If the coalition defeated D and divided the pie peacefully at time $t = 1$, A offers B a share $z_2 \in [0,1]$, which B can accept or reject. Finally, if one partner defeated the other in an intra-coalition war at $t = 1$, D proposes $a_2, b_2 \in [0,1]$ to divide the total pie with the indicated surviving coalition member, which leads to settlement or war.

War is a costly lottery. The winning side eliminates the losing side, but fighting destroys a fraction $d \in (0,1)$ of the stakes. If D defeats the coalition, for example, its payoff for $t = 1$ is $1 \cdot (1-d) = 1-d$, and it enjoys only $1-d$ at $t = 2$. If a coalition member wins an intra-coalition war after accepting a proposal from D, it gets $x_t(1-d)$ for the period, and any subsequent war destroys another share of the stakes in question. War outcomes are a function of relative military capabilities, which depend on initial endowments and possible increases in D's and B's relative capabilities. First, D's capabilities may increase relative to the coalition between the first and second periods if the war ends, reflecting operational advantages enabled by a pause, the completion of armament programs, or the consolidation of wartime gains. Second, B's capabilities may increase relative to A if the fighting continues. B may take on larger shares of the war effort, consolidate control over strategic territory, increase mobilization and productive capacity over time, or see its partner's capabilities fall due to military exhaustion or a collapsed domestic consensus. While it's possible for each of these factors to either boost or reduce the coalition's chances of defeating D in a continued war, I abstract away from them in the main analysis. As I show in the appendix, this additional source of shifting across-side power only exacerbates the model's commitment problems; it doesn't alter *why* war ends early or late, only just how early or late the war ends.

Two parameters summarize the effects of shifting power. First, the coalition defeats D in a war with probability $p_C \in (0,1)$, which can take on two values,

$$p_C = \begin{cases} p_C^m & \forall\, t \text{ if power is static} \\ p_C^l & \text{at } t = 2 \text{ if } D \text{ has grown stronger,} \end{cases}$$

where $p_C^m > p_C^l$ such that the coalition's chances of victory are greatest at p_C^m and lowest when p_C^l. D loses with the complementary probability. The game begins at p_C^m, the initial distribution of power from which the coalition falls to p_C^l if power shifts in D's favor. I save a consideration of p_C^h, where fighting may also increase the coalition's relative power, for the

[5]Given the model's focus on explaining war termination and continuation at time $t = 1$, assuming a $t = 2$ war to be decisive is without loss of generality. It's important only that second-period outcomes depend on players' relative power.

appendix. Second, B defeats A with probability $p_B \in (0,1)$,

$$p_B = \begin{cases} p_B^l & \forall\, t \text{ if power is static} \\ p_B^h & \forall\, t \text{ if } B \text{ has grown stronger,} \end{cases}$$

where $p_B^h > p_B^l$, and where A loses with the complementary probability. When intra-coalition power is static, B's chances of defeating A in an intra-coalition war are p_B^l regardless of whether the war continues or ends, but when fighting will shift power in B's favor, its chances of defeating its partner rise to p_B^h.

Finally, the model represents cases in which power is static and shifting across and within sides. The variable $s(D,B)$ summarizes four states of the world, such that

$$s(D,B) = \begin{cases} s(0,0) & \text{power is static} \\ s(0,1) & B \text{ is rising, } D \text{ is not} \\ s(1,0) & D \text{ is rising, } B \text{ is not} \\ s(1,1) & D \text{ and } B \text{ rising.} \end{cases}$$

Across-side power is static in cases $s(0,B)$ and shifting in cases $s(1,B)$, while within-side power is static in cases $s(D,0)$ and shifting in cases $s(D,1)$. The war termination literature covers differences between states $s(0,0)$ and $s(1,0)$, with shifting across-side power (cf. Leventoğlu and Slantchev 2007, Powell 2012, Wolford, Reiter and Carrubba 2011). I show next, however, that comparing states $s(0,0)$ to $s(0,1)$ and $s(1,0)$ to $s(1,1)$ shows how coalition politics can explain on-time, early, and late war terminations in a single model.

Analysis

I solve for Subgame Perfect Equilibria (SPE), in which strategy profiles entail Nash Equilibria in every proper subgame. SPE rules out promises that players have no incentive to keep, which makes it useful for commitment problems: D can't promise not to demand more of the coalition it'll grow stronger in the future, nor can B promise not to demand a larger share of the pie after a military victory that increases its strength relative to A. Before discussing equilibria, I first link D's potential increase in relative power to the pre-play course of the war. Then, I impose restrictions on two-player settlements, simplifying the analysis at no cost to the substantive character of the results.

Suppose that war between C and D began due to the latter's prospective rise in power. Continuing the war can produce a decisive outcome, but fighting is costly, and absent coalition politics there's no incentive to continue the war. Therefore, a war that continues in the first period despite static across-side power ends *late*. In states $s(1,B)$, D's commitment problem is unsolved when the game begins; if the war ends with a negotiated settlement, D can renegotiate from a position of greater relative power at $t = 2$. In the absence of the credible pre-commitments ruled out by the structure of the international system, the only way for the coalition to lock in a distribution of benefits based on D's first-period capabilities is to continue fighting. Therefore, if a war ends in the first period that would have continued under static intra-coalition power, it ends *early*.

Next, it's useful to make some assumptions about bilateral bargains, i.e. all bargains except D's proposals to the coalition. None of these negotiations (y_t, w_t, z_2, a_2, b_2) occur in the shadow of shifting power or private information, so it's easy to show that they're struck peacefully. These wars stay off the equilibrium path, but specifying a bargaining protocol introduces unnecessary parameters. Therefore, I assume that all bargains $\neg x_t$ reflect relative military power (cf. Debs and Goemans 2010), which is consistent with countries' freedom to resort to force in an anarchic international system and focuses the analysis on show shifting relative power affects war termination. If war ends or if C defeats D at $t = 1$, then $y_1, w_1 = p_B$. This makes it easier to analyze D's proposals x_t, which entail explicit coalition politics and bear directly on war termination.

I first characterize general features of coalition politics when A and B respond to D's proposals, highlighting the role of the costs of discord in determining which partner determines war aims. Then, I divide the equilibrium analysis into several sections, imposing restrictions at different points to isolate specific causal mechanisms (see Paine and Tyson 2020). In states $s(0,B)$, where war has already solved the underlying bargaining problem, war ends late when the costs of discord are sufficiently low and B's rise in power is sufficiently large. In states $s(1,B)$, where war has yet to solve the underlying bargaining problem, war ends early when the costs of discord are sufficiently high and and B's rise is sufficiently large. I discuss the late end of the Paraguayan War briefly in this section, saving an extended discussion of the early end of the First World War for its own section.

Coalition Politics

Coalition politics shapes war termination similarly in each equilibrium, and in this section I summarize its effects as a function of (a) shifting intra-coalition power and (b) the costs of coalition discord. Recall that A can continue the war if it wishes, which gives it some agenda-setting power, but that B can force continuation at the cost of coalition discord. In equilibrium, either member may set x_t^C, the coalition's *acceptance constraint*, which defines the minimum offer that D must make to ensure peace. The acceptance constraint is a function of partners' minimum demands for peace, x_t^A and x_t^B, such that when given a choice, a coalition member prefers all offers at least as large as its minimum demand to war ($x_t \geq x_t^{A,B}$) and prefers war to any smaller offer ($x_t < x_t^{A,B}$). Formally, the coalition's acceptance constraint is $x_t^C = \max\{x_t^A, x_T^B\}$. Lemmas 1–3 establish how the acceptance constraint is defined in any equilibrium, beginning with the effects of B's threat to continue the war on A's willingness to reject D's proposals.

Lemma 1. *A rejects any proposal that B will reject.*

Lemma 1 establishes that A rejects any proposal x_t that B would reject if given the chance. Overt coalition disagreement is mutually costly, so A avoids it, preferring an agreed war to one brought about via B's veto. A can have the wars it wants, but A will also grant B the wars it wants if the alternative is open discord. Lemma 1 also implies that the costs of discord are never paid in equilibrium, but as stated in Lemmas 2 and 3, they determine who sets the acceptance constraint and the terms on which D can end to the war.

Lemma 2. *In states $s(D,0)$, $x_t^A > x_t^B$ such that A sets the acceptance constraint.*

Lemma 2 describes states $s(D,0)$, where intra-coalition power is static. Each coalition member gets the same share of the pie after a settlement (y_t) and military victory (w_t), which ensures that the only difference in their minimum demands is the costs of discord that B would impose by rejecting. Those costs, however, ensure that A rejects a wider range of proposals than B, or $x_t^A > x_t^B$. Therefore, if A accepts a proposal, B is sure to as well, since A can demand more in lieu of war than B. Therefore, when intra-coalition power is static, the coalition finds it easy to agree on ending or continuing the war.

Lemma 3. *In states $s(D,1)$, $x_1^A \geq x_1^B$ such that A sets the acceptance constraint when*

$$c \geq \frac{p_B^h - p_B^l}{p_B^h\left(1-p_B^l\right)} = \hat{c}. \tag{1}$$

Otherwise, $x_1^A < x_1^B$ such that B sets the acceptance constraint.

Finally, when power is shifting between coalition partners, the terms of within-side bargains depend on whether the war continues. Accepting D's proposal in the first period fixes B's power relative to A's at p_B^l, but rejection forces a fight to the finish and sees B's power rise to p_B^h. B's rise confronts A with a dilemma. Continuing may secure the whole pie from D, but it also entails yielding a larger share of the intra-coalition pie to B; likewise, ending the war prevents B's rise but leaves a smaller pie to be shared. For its part, B gains more from continued war than A—because $p_B^h > p_B^l$—and absent the costs of discord it would reject more generous proposals than A. Lemma 3 states that, when overturning A's acceptance is sufficiently costly ($c \geq \hat{c}$), B's minimal demands are no more aggressive than A's, or $x_1^A \geq x_1^B$. As such, A sets the acceptance constraint. On the other hand, if coalition discord isn't too costly ($c < \hat{c}$), B sets the acceptance constraint, forcing A to reject proposals that it would otherwise accept (i.e., when $x_t^A < x_t < x_t^B$). Line (1) in Lemma 3 also shows that $\hat{c}$ increases in B's post-shift relative power, p_B^h, such that B is more likely to set the acceptance constraint as its prospective boost in power grows larger. Therefore, whether B can set a high acceptance constraint against A's wishes depends on both the costs of discord and the extent to which B is strengthened by fighting.

At this point it's useful to define two ratios,

$$\alpha = \frac{1-p_B^h}{1-p_B^l} \quad \text{and} \quad \beta = \frac{p_B^h}{p_B^l}, \tag{2}$$

which summarize the effect of continuing the war on relative power inside the coalition. When α is close to one, the difference $p_B^h = p_B^l$ is small, and continued fighting has a limited impact on relative power; but as α decreases, fighting shifts relative power farther in B's favor, such that A prefers larger values of α. And when β is close to one, continued fighting has little impact on B's power relative to A, but as it increases, continued fighting promises ever larger shares of the intra-coalition pie, such that B prefers larger values of β.

Lemma 3 has an additional implication for the terms on which coalition wars end. When B can credibly threaten to continue the war, the acceptance constraint ($x_1^C = x_1^B$) is a function of the costs of discord. The coalition avoids overt disagreement, but since B forces a continuation of the war by threatening a public breach, the costs of discord are reflected in the acceptance constraint. Therefore, as the costs of discord increase, D can extract better terms in intra-war negotiations, even though an intra-coalition breach doesn't occur in equilibrium. In other words, wedge strategies (see Crawford 2011) need not actually split the coalition to pay dividends. This relationship only arises, however, when intra-coalition power is shifting; when there are no coalition bargaining frictions to exploit, D must make more generous offers to secure an end to the war. Even when bargaining frictions don't undermine the coalition's cooperation, its opponent can take advantage of them to secure better deals at the negotiating table.

Problem Solved

This section considers cases $s(0,B)$, where fighting has already solved the war's underlying bargaining problem. D's rise has been arrested, so it can credibly commit to agreements reflecting the initial distribution of power. This ensures an on-time end to the war. But when intra-coalition power is shifting, B may force a fight to the finish because it can't be compensated in peace for the foregone benefits of increased strength.

Proposition 1. *In state $s(0,0)$, war ends at $t=1$.*

Proposition 1 describes state $s(0,0)$, a frictionless bargaining environment in which the pregame course of fighting has stabilized the across-side distribution of power, giving belligerents no incentive to continue the war at $t=1$ or to fight a new one at $t=2$. By Lemmas 1 and 2, we know that A sets the acceptance constraint, which reflects the coalition's chances of victory in continued war (p_C^m). And since further fighting is costly, D meets the acceptance constraint at equality, which guarantees acceptance. War ends on time, as anticipated by dyadic models of endogenous war termination.

Proposition 2. *In state $s(0,1)$, war ends at $t=1$ when A sets the acceptance constraint ($c \geq \hat{c}$). When B sets the acceptance constraint ($c < \hat{c}$), war continues if either*

$$\left(1-p_C^m\left(\beta\right)(1-d)(1-c)\right)\frac{1}{1-p_C^m} < 1-d \tag{3}$$

such that D prefers war to meeting B's minimum demands ($x_1^* = x_1^B$), or

$$p_C^m\left(\beta\right)(1-c)(1+\delta)-p_C^m\delta > \frac{1}{1-d} \tag{4}$$

such that $x_1^B > 1$. Otherwise, the war ends.

Proposition 2, which covers state $s(0,1)$, describes equilibria in which shifting intra-coalition power complicates the simple decisions of state $s(0,0)$. Peace is again assured in the second period, and since D isn't rising, there's no incentive to continue the war to shape the across-side distribution of power. Now consider first-period decisions about ending the war. If the war ends, A and B strike a series of intra-coalition bargains that reflect B's initial probability of victory, $y_t = p_B^l$. If the war continues, however, B's relative power increases over the course of the fighting, and the coalition strikes postwar bargains around

B's increased power, $w_1 = x_1 = p_B^h$. This gives B an incentive to continue fighting despite the costs of war. If the war ends on time, A can't credibly promise to compensate B for the foregone increase in its capabilities, so the only way B can secure an increased share of the pie is to continue the war despite having solved D's commitment problem.

Country B may stand more to gain from continuing the war than A, but Proposition 2 shows that the war need not end late. First, only when power will shift sufficiently in its favor does B wish to continue the war. Lines 3 and 4 show when, if given the chance, B will set such an aggressive acceptance constraint that it outstrips either what D is willing to concede or, when $x_1^B > 1$, the per-period size of the pie. Each condition becomes easier to satisfy with increases in B's prospective growth in power, β as defined by Line (2); the more that continued fighting increases B's share of the intra-coalition pie, the more likely are its minimum demands to preclude an on-time end to the war. Second, B can only ensure the continuation of the war when the costs of coalition discord are sufficiently low, or $c < \hat{c}$. If $c \geq \hat{c}$ such that A sets the coalition's acceptance constraint, B is restrained and the war ends at $t = 1$. Therefore, shifting power and coalition politics determines whether the war continues; when the costs of discord are high, the war ends on time, but when the costs of discord are low and fighting boosts B's power sufficiently, the war ends late.

This equilibrium offers a plausible account of the termination of the Paraguayan War, which ended with the death of Paraguay's leader, Javier Solano López, well after he'd lost control of the capital, the government, and most of Paraguay. The war could plausibly have ended in 1869 (Strauss 1978, 21), yet it continued for another bloody year, as Brazil insisted on a total military victory over Argentina's preference for negotiating. One reason for this difference in aims was Brazil's relatively larger share of the war effort and control of key rivers, which would ensure not only that it could dominate the occupation but that it could dictate terms to both Paraguay *and* Argentina (Warren 1972, 388-390). Brazil's continuation of the war entailed coalition discord—including postwar crises with its erstwhile partner (see Strauss 1978)—and, consistent with the equilibrium, Brazil managed to force Argentina to accept less territory than it had been promised in the Treaty of Triple Alliance at the outset of the war. The equilibrium also offers an alternative to Weisiger's (2013) account of the duration of the war, which relies on faulty Brazilian and Paraguayan inferences about each other's goals in the conflict; such uncertainty may have been present, but the intra-coalition rivalry between Brazil and Argentina can account for the former's more extensive war aims *and* its willingness to limit the latter's postwar territorial gains.

Problem Unsolved

This section considers states $s(1,B)$, where fighting has yet to solve the war's underlying bargaining problem. Two-player models expect the war to continue as long as power is still shifting sufficiently in D's favor, which helps establish a baseline for an on-time end to the war when intra-coalition power is static. I show that shifting power inside the coalition can induce B to continue the fight under conditions that would ensure peace in two-player models, but war can also end sooner than it would otherwise when the costs of coalition discord are sufficiently high.

Proposition 3. *Let $p_C^h = p_C^m$. In state $s(1,0)$, war continues at $t = 1$ when*

$$p_C^m(1+\delta) - p_C^l\delta > \frac{1}{1-d} \tag{5}$$

such that $x_A^1 > 1$, and ends otherwise.

Proposition 3 describes equilibrium outcome in state $s(1,0)$. Fighting has yet to prevent D's prospective rise, but intra-coalition power remains static. Peaceful outcomes in the second period are assured, since power is no longer shifting, and by Lemma 2, we know that A sets the the coalition's acceptance constraint. And since D's commitment problem remains unsolved, Proposition 3 gives a result similar to standard treatments of the commitment problem (see Powell 2006, 181-183). Line (5) show that the war continues when the size of the prospective shift in power against the coalition is larger than the costs of a fight to the finish. D's rise will allow it to demand so much in the future that the coalition's required compensation outstrips the size of the pie ($x_A^1 > 1$), and the coalition continues the war to a decisive end. Proposition 3 also sets a baseline for war termination. When Line (5) is satisfied, the war ends on time at time $t = 1$ if it continues but early it ends; when Line (5) isn't satisfied, the war ends on time if it ends at $t = 1$ but late if it continues.

Proposition 4. *Let $p_C^h = p_C^m$. In state $s(1,1)$, war continues at $t = 1$ when A sets the acceptance constraint ($c \geq \hat{c}$) and*

$$p_C^m(\alpha)(1+\delta) - p_C^l\delta > \frac{1}{1-d} \tag{6}$$

such that $x_A^1 > 1$. When B sets the acceptance constraint ($c < \hat{c}$), war continues if either

$$\left(1 - p_C^m\left(\beta\right)(1-d)(1-c)\right)\frac{1}{1-p_C^m} < 1-d \tag{7}$$

such that D prefers war to meeting B's minimum demands ($x^* = x_B^1$), or

$$p_C^m\left(\beta\right)(1+\delta) - p_C^l\delta > \frac{1}{1-d} \tag{8}$$

such that $x_1^B > 1$. Otherwise, the war ends.

Proposition 4 describes equilibrium outcomes in state $s(1,1)$. Power is shifting both across *and* within sides, which confronts A with a dilemma. Solving one rising country's commitment problem activates the other's; continuing the war ensures a more favorable across-side bargain but a less-favorable within-side bargain, but stopping the war preserves a more favorable within-side bargain at the expense of an eroding across-side bargain. The rising D is willing to make generous offers to avoid continued war, and A is receptive to some, but not all of them. When B sets the acceptance constraint, however, its minimum demands may again be so large that D is unwilling or unable to meet them, ensuring that war lasts longer than it would otherwise. And intra-coalition politics determines which partner sets the acceptance constraint and, therefore, when the war ends.

First, when the costs of discord are low ($c < \hat{c}$), B sets the acceptance constraint. The promise of increasing its share of the pie relative to its partner renders B willing to tolerate the costs of war under conditions that it wouldn't in state $s(1,0)$, again because power is shifting such that B demands more than either D will or can yield (i.e., $x_1^B > 1$). This drives B to continue the war under conditions that see the war end in Proposition 3. To see this, compare the left sides of the war conditions in Lines (5) and (8), since their right sides are identical,

$$\underbrace{p_C^m(1+\delta) - p_C^l\delta}_{s(1,0)} < \underbrace{p_C^m\left(\beta\right)(1+\delta) - p_C^l\delta}_{s(1,1),\ c<\hat{c}},$$

where the left side is less than the right, since B stands to gain distributively from continued fighting ($\beta > 1$). The larger B's prospective increase in relative power, i.e. as β increases, the more willing B is to force a continuation of the war under conditions that would end the war under static intra-coalition power. Therefore, when power is shifting in B's favor and the costs of coalition discord are low, the war ends late.

Second, when the costs of discord are high enough to ensure that A sets the acceptance constraint ($c \geq \hat{c}$), coalition politics produces an early end to the war, B's rising power notwithstanding. Comparing the left sides of the war conditions in Lines (5) and (6),

$$\underbrace{p_C^m(1+\delta) - p_C^l\delta}_{s(1,0)} > \underbrace{p_C^m(\alpha)(1+\delta) - p_C^l\delta}_{s(1,1),\ c\geq\hat{c}}, \tag{9}$$

shows that the left side is greater than the right, since $\alpha < 1$. A's prospective share of the pie (α) falls as B's prospective advantage over A grows, making A still more willing to make peace with a rising enemy. Restraining B's desire to continue the war, however, comes at a cost. As shown in Line (9), A's lower threshold for ending the war means that it tolerates some across-side shifts in power that would prompt it to continue the war if intra-coalition power were static. Rather than solve D's commitment problem, A chooses to solve B's, ending the war early and leaving the bargaining problem that motivated the war in the first place unsolved. The model thus accounts for early terminations without reference to collective action, battlefield realities, exhaustion, or third-party pressure. The belligerents end the war themselves, knowingly leaving the war's underlying problem unsolved, the enemy relieved and the coalition at odds. I show below that this equilibrium sheds light on the end of the First World War, whose eventual victors had expected to fight on rather than accept an armistice in the fall of 1918.

The Armistice of 1918

When the First World War ended with an armistice on the Western Front in 1918, German territory remained unconquered, its army was mostly intact, and its top generals envisioned resuming the war if Allied demands proved excessive. Armistice was at clear variance with Allied rhetoric and war planning in the years and months before, which envisioned a decisive drive into Germany.[6] Britain's Prime Minister David Lloyd George had declared that

[6]A note on terminology: I refer to Britain and France as the Entente, but when combined with the United States, which joined the was as an "Associated" power, I follow convention and call this grouping the "Allies."

"The fight must be to the finish—to a knockout" (quoted in Meyer 2016, 153). French premier Georges Clemenceau declared as late as September 1918 that "[a] military decision is what Germany desired and has condemned us to pursue" (quoted in Stevenson 1982, 114). Even after granting the armistice, American President Woodrow Wilson, who had previously talked of "marching triumphantly into Berlin," remained skeptical that Germany's new civilian government would be able to honor the peace (Kennedy 2001, 19-20). Germany's enemies believed that the Kaiser and his generals were bent on unhinging the global balance of power (see Hull 2014, 271) and that they would be tempted to take advantage of a pause to resume the fighting. The best solution seemed to be a fight to the finish, a "knockout" that would disarm Germany and disempower its generals. Why, even as Allied war plans accounted for fighting into 1919 (French 2002, Ch. 10, McCrae 2019), and American dollars, dreadnoughts, and doughboys poured into Europe to tip the scales against the Central Powers, did the Allies settle for armistice in Belgium instead of victory in Germany? Why did the First World War end "early"?

Answering this question requires showing that the model can explain the Allied decision to grant Germany's request for armistice, even though continued fighting promised a more robust solution to the underlying commitment problem. The Entente and the United States were both mindful of the costs of fighting, but that doesn't explain why the Entente would settle for an armistice rather than make a German return to war more difficult—say, by forcing surrender and creating buffer states on the Rhine (Stevenson 2005, 124-125)—and why the rising United States, aware of increasing Entente dependence on its military power (*ibid.* 112), wasn't more eager to fight on to the finish and establish an American peace. This requires treating the entire equilibrium, which depends on both the game form and the solution concept, as the causal mechanism (Goemans and Spaniel 2016). The analyst must show that (a) the structure of interactions to be explained is similar to the game form, (b) the facts of the case match the parameter values that support the existence of the relevant equilibrium, and (c) decision-makers reasoned as actors do in equilibrium, particularly with respect to what they believed would be the consequences of choosing other than they did. To that end, I provide evidence that (a) the Allies worried that armistice would allow Germany to resume fighting, (b) the Allies believed that a longer war would increase America's influence over the settlement, (c) the Entente powers accepted Germany's proposed armistice to limit American influence over the peace, and (d) the Americans accepted the same early end thanks in part to maintaining Allied unity.

First, the game form represents essential features of the case. In late 1918, the Allies and Germany weighed continued war against an armistice, which would pause the fighting to allow for negotiations. Ending the war short of a total Allied victory would fix the prevailing military situation for the time being, and though Germany knew that its ability to fight was almost extinguished, the Allies were mostly unaware of the scale of their victory (Stevenson 2005, 108). The prospect of armistice tempted Germany's Quartermaster General Erich Ludendroff with the chance to "regroup at his own pace and carry out an orderly retreat, re-opening a defensive battle on a shorter line if Allied demands were excessive" (Stevenson 2005, 120). Indeed, the German High Command hoped to use troops evacuated from Belgium and France "to re-establish the the military status quo of 1914 for the duration of the peace negotiations" to preserve the option to resume the war from a strong defensive position if talks failed (Gerwarth 2020, 71). On the Allied side, Britain's War Cabinet worried in fall 1917 that a settlement that reflected German gains in France, Belgium, and Eastern Europe would allow it to grow stronger and resume its attempts to dominate Europe, "making a limited war outcome unstable" (Reiter 2009, 173). And French "President Raymond Poincaré... wrote that a ceasefire would be a 'trap,' allowing the enemy to shorten their lines and 'cutting the hamstrings of our solders'" (*ibid.*, 120). The French weren't alone; Lloyd George and American General John Pershing both expressed similar worries about the effect of an armistice on Allied morale in October (Lowry 1996, 23,96). Therefore, an armistice might allow a relatively strengthened Germany to undermine the initial terms of settlement, as the coalition's relative strength fell from p_C^m to p_C^l. The war's underlying commitment problem remained unsolved, placing the fall of 1918 in states $s(1,B)$.

If armistice might've enabled a German resurgence, continuing the war would've dramatically increased American power relative to its Entente partners. First, the Allies expected that (a) the war would go on into 1919, thanks to Germany's ability to exploit Russia's surrender at Brest-Litovsk, and (b) American manpower, up to 100 divisions by some estimates, would be necessary to ensure victory (McCrae 2019, Ch. 5). But in summer 1918, barely a third of that number was in the field. In October, Jan Smuts argued in the Imperial War Cabinet that "Britain's relative strength would only decline from now on" if the war continued (Stevenson 2005, 124). And as the American war effort spun up to capacity,

> Clemenceau confided to Lloyd George his fear that if America supplied too many of the effectives on the Western Front it would be able to decide the outlines of the settlement (Stevenson 1982, 110).

The Americans knew that they held the ring and that their grip would grow only tighter. Wilson wrote to his personal envoy Edward House that "When the war is over we can force [Britain and France] to our way of thinking because by that time they will, among other things, be financially in our hands" (quoted in Mayer 1969, 332).[7] As early as 1917, Wilson had been content "to let the fighting run on, so that the Allies would become more dependent on the US, its own war effort could expand, and it could wield more leverage over the peace" (Stevenson 2005, 112). Finally, American soldiers outnumbered British by early November and would in a matter of months outnumber French forces as well (*ibid.*, 131). Rising American power inside the coalition was commonly known and, in the case of the British slowing the flow of doughboys to France (Parsons 1977), resisted once victory looked likely, as coalition members vied to increase their shares of the pie at each other's expense (Goemans 2000, 291). Thus, the end of World War I falls squarely in state $s(1,1)$, where power is shifting both across and within sides.

Next, the model's representation of coalition politics reflects distributive disagreements among the Allies and awareness of the costs of discord. Coalition disagreed over how to share the fruits of victory, which amounted to a say in designing a new international legal-political regime (Fischer 1967, Hull 2014, Parsons 1978, Rothwell 1971, Stevenson 1982), the scope of which had been increased by Lenin's and Wilson's competing appeals to overturn the imperial system (see Manela 2009). Britain and France wished to expand their empires, the Americans to limit them, and the means by which each could pursue those ends depended on influence over the peace settlement. Germany recognized these divergences, pointedly making its first armistice appeal to the United States (Herwig 2014, 411).

[7] I owe my awareness of this quote to Goemans (2000), who found it in Stevenson's (1982) footnotes.

The Allies wrangled over whether the Fourteen Points, a set of vague principles on which both France (Stevenson 1982, Ch. IV) and Britain (Rothwell 1971, Ch. VI) looked with scorn, could be accepted as the basis of negotiations. The Allies nonetheless wished to avoid outright disagreement, which might (a) embolden Germany or make it more difficult to deter, (b) establish a reputation for weakness in postwar negotiations, or (c) compromise the pursuit of other postwar goals, which by 1918 included "combat[ting] the spread of Bolshevism in wake of the Russian Revolution" (McCrae 2019, 243) and managing the dissolution of the Ottoman Empire (Fromkin 1989). Therefore, the model represents both the distributive disagreement between the Americans and the Entente over the postwar pie and a shared awareness of the costs of a public breach.

Explaining why the war ended in an armistice and not a drive into Germany requires explaining why the Entente and the United States were willing to forego military victory, when (a) the former knew that it offered better guarantees against revanchism and (b) the latter knew that continuing the war would secure greater influence over the postwar world. Coalition politics provides an answer. Jan Smuts drew a distinction between the "British peace" made possible by an armistice and the "American peace" that would follow victory in 1919, (Goemans 2000, 292-294) and "[o]n 26 October, finally, the War Cabinet agreed that Britain would get a better peace in 1918 than in 1919" (*ibid.*, 293). This was no small sacrifice, but Britain saw in American dominance a prohibition on the right of blockade, a bedrock of British grand strategy (Rothwell 1971). Better to end the war on imperfect terms than let the Americans carry the day and gain the political weight to rewrite the laws of the sea. France saw things similarly. Clemenceau and Marshal Ferdinand Foch knew that military victory—which would include an immediate occupation of the Rhineland—might have offered a more permanent answer to the German question, but they "had no particular interest in going on to Berlin if it would weaken France and strengthen America's relative position" (Stevenson 2005, 125, 127). Better to end the war in 1918 than to make France more vulnerable to American predominance.

Finally, what of the United States, the coalition partner that stood the most to gain from taking the war into Germany? Wilson's desire to reshape the world required humbling the European powers in whose favor he'd tilted the balance in 1917 (see Tooze 2014). In Proposition 4's equilibrium, the rising partner yields to the declining partner's desire to end the war early because the costs of discord loom larger than the distributive gains of further fighting, i.e. when $c \geq \hat{c}$. Similar concerns about maintaining Allied unity guided American support for the armistice in 1918. First, recognizing that threats to return to war were a critical backstop in securing advantageous terms (Lowry 1996, 24), Wilson took several steps to avoid a public breach with the Entente, including a studied refusal to give a formal interpretation of the Fourteen Points (31) and promises not to act unilaterally in imposing terms on Germany (34). Second, Wilson also feared that continuing the war would undermine progressive politicians in Britain and France—to say nothing of his own domestic agenda—all of which might make it more difficult to get the terms he wanted even after a total victory (Stevenson 2005, 120-122). With the British and French mostly supportive of a vaguely-construed Fourteen Points as a basis for negotiating peace, Wilson was willing to forego further fighting. Finally, ending the war in 1918 served an additional collective goal that open discord might have compromised: cooperation in the face of the perceived threat of Bolshevism spreading outward from Russia (McCrae 2019, 243). The

Figure 3: Shifting power, the costs of coalition discord, and war termination

Within-side power ↓ / Across-side power →	Static	Shifting
Static	$s(0,0)$ on time	$s(1,0)$ on time
Shifting	$s(0,1)$ late	$s(1,1)$ late if $c < \hat{c}$ early if $c \geq \hat{c}$

costs of discord loomed large for the Allies in 1918, and though they harbored doubts about the credibility German commitments to peace, they agreed to grant the armistice rather than see within-side power shift dramatically across the Atlantic.

Coalition politics offers a useful explanation for the "early" end of the First World War on the Western Front. Germany, its military exhausted and almost mutinous, requested an armistice that preserved a chance of breakout that eventual defeat would foreclose. The British and French, faced with the tradeoff of breaking German power at the cost of American dominance in Europe, were inclined to a settlement that, all else equal, they might have rejected in favor of a drive into Germany. The Allies weren't yet aware of the scale of their victory—the armistice would soon turn into an effective surrender—and the prospect of resumed hostilities was all too real. Yet as hard as intra-coalition bargaining over the terms of armistice was, with threats of separate peaces lobbed across those partners that had and hadn't signed the Declaration of London (French 2002, Rapp Hooper 2014), it would've been harder still for the British and the French to secure their aims had the war continued into 1919. They might've traded a relatively favorable European settlement for an unambiguously American one built around Wilson's desire to end Europe's imperial system. Finally, the United States, putative beneficiary of a military victory, granted the armistice because an open breach with its partners would've been worse. Had Wilson been able to pursue his power-political goals in Europe without Entente cooperation, the war might have ended not in 1918's armistice but in a drive into Germany.

Conclusion

Relaxing the two-player assumption standard in models of war termination can improve our understanding of how wars end. Contrary to accounts based on collective action or coalition size, coalition politics can increase *or* decrease the duration of war when power is shifting between partners. Figure 3 summarizes the model's implications for the timing of war termination, which depend on the interaction of shifting power and the costs of coalition discord. When across-side power is static, wars end late when continued fighting promises to increase one coalition partner's share of the postwar pie. When across-side power is shifting, shifting power inside the coalition can shorten the war, leaving the underlying bargaining problem unsolved, when the costs of a public coalition disagreement are sufficiently high. Therefore, intra-coalition politics can account for wars that end on time, early, and late without reference to collective action or coalition size. The model also illuminates why the First World War ended short of the long-contemplated drive into German territory.

Modeling coalition politics can improve the empirical and theoretical analysis of war duration. Variables like coalition participation (see Morey 2016, 2020), within-side shifts in wartime power, and the costs of discord can help sort through conflicting estimates of the relationship between the number of belligerents on war duration. Coalition politics, as distinct from the number of parties to the conflict, can either lengthen or shorten wars; the present model anticipates no consistent bivariate relationship. There are also several paths for future theoretical development. First, though the search for credible commitments to the terms of peace can bedevil wars begun by any bargaining problem (see Beard 2019, Goemans 2000, Reiter 2009), future work can explore wars caused by information problems. Results shouldn't differ in the case where the problem has been solved, as described by Propositions 1 and 2, but it's less clear how unsolved information problems—say, uncertainty over the military balance (Slantchev 2003), resolve (Powell 2004), or ideal policies (Spaniel and Bils 2018)—might interact with intra-coalition politics to shape war termination. Future work might also explore (a) how bargaining frictions associated with other explanations, like political survival incentives, shape intra-coalition politics, and (b) richer models of coalition politics than the spare one presented here. For example, power shifts between partners exogenously in this model, but future work might model choices that shift power (cf. Debs and Monteiro 2014), like the Entente's decision to lash itself financially to the United States during the First World War, which ensured military superiority over the Central Powers but risked turning over the postwar order to the Americans.

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Appendix

Proofs

Proof of Lemma 1. Letting $u_A(\text{reject}_t|x_t)$ denote A's payoff for rejecting a proposal x_t. Accepting a proposal that B is sure to reject yields $u_A(\text{reject}_t)(1-c)$, which leaves A strictly worse off than rejecting. □

Proof of Lemma 2. I focus on $t=1$, since the logic carries straightforwardly over to $t=2$. Given $s(D,0)$, B accepts some x_1 iff

$$x_1 \cdot p_B^l + \delta x_2^* p_B^l \geq p_C^m p_B^l (1-d)(1-c)(1+\delta),$$

or

$$x_1 \geq p_C^m (1-d)(1-c)(1+\delta) - \delta x_2^* = x_1^B,$$

and A accepts iff

$$x_1(1-p_B^l)+\delta x_2^*(1-p_B^l) \geq p_C^m(1-p_B^l)(1-d)(1+\delta),$$

or when

$$x_1 \geq p_C^m(1-d)(1+\delta)-\delta x_2^* = x_1^A,$$

where $x_1^A > x_1^B$ is ensured by $0 < c < 1$. □

Proof of Lemma 3. I focus on $t=1$, since power is no longer shifting between coalition partners at $t=1$. In states $s(D,1)$, B accepts iff

$$x_1 p_B^l + \delta x_2^* p_B^l \geq p_C^m p_B^h(1-d)(1-c)(1+\delta)$$

or

$$x_1 \geq p_C^m\left(\frac{p_B^h}{p_B^l}\right)(1-d)(1-c)(1+\delta)-\delta x_2^* = x_1^B.$$

By similar logic, A accepts iff

$$x_1 \geq p_C^m\left(\frac{1-p_B^h}{1-p_B^l}\right)(1-d)(1+\delta)-\delta x_2^* = x_1^B.$$

Finally, $x_1^A \geq x_1^B$ when

$$c \geq \frac{p_B^h-p_B^l}{p_B^h\left(1-p_B^l\right)} = \hat{c},$$

and $x_1^A < x_1^B$ when $c < \hat{c}$. □

Proof of Proposition 1. Since two-player bargains are all based on the distribution of power and, by Lemma 2, A sets the acceptance constraint, I begin with A's choice of accepting or rejecting x_2, which B is also sure to accept. A accepts iff

$$x_2(1-p_B^l) \geq p_C^m(1-p_B^l)(1-d)$$

or

$$x_2 \geq p_C^m(1-d) = x_2^A. \tag{10}$$

Next, if D wishes to induce acceptance it meets A's acceptance constraint at equality $x_2 = x_2^A$, since any larger offer would leave D worse off. Finally, D proposes $x_2^* = x_2^A$ rather than a smaller offer that induces rejection when

$$1-p_C^m(1-d) \geq (1-p_C^m)(1-d),$$

which is sure to be true since $0 < d < 1$. Next, consider A's decision at $t = 1$, where it again sets the acceptance constraint. A accepts iff

$$x_1(1-p_B^l)+\delta x_2^*(1-p_B^l) \geq p_C^m(1-p_B^l)(1-d)(1+\delta),$$

or when

$$x_1 \geq p_C^m(1-d) = x_1^A.$$

Again, D meets the acceptance constraint at equality, $x_1^* = x_1^A$, if it wishes to induce acceptance, and it does so when

$$1-p_C^m(1-d)+\delta(1-x_2^*) \geq (1-p_C^m)(1-d)(1+\delta),$$

which is sure to be the case, given $0 < d < 1$. □

Proof of Proposition 2. Since two-player bargains are all based on the distribution of power, I focus only on across-side offers x_t. First, let $c \geq \hat{c}$ such that A sets the acceptance constraint. At time $t = 2$, A accepts iff $x_2 \geq p_C^m(1-d) = x_2^A$ as defined in Line (10), which given the absence of shifting power D is sure to meet at equality. Moving to the first period, A accepts iff

$$x_1(1-p_B^l)+\delta(1-x_2^*)(1-p_B^l) \geq p_C^m(1-p_B^h)(1-d)(1+\delta),$$

or

$$x_1 \geq \frac{p_C^m(1+\delta p_B^l-(1+\delta)p_B^h)(1-d)}{1-p_B^l} = x_1^A.$$

This value is sure to be less than one, such that A never demands more than the per-period value of the pie, and if D wishes to induce acceptance it will meet the constraint at equality. And D proposes $x_1^* = x_1^A$ rather than some smaller offer sure to be rejected when

$$1-x_1^A+\delta(1-x_2^A) \geq (1-p_C^m)(1-d)(1+\delta),$$

which is sure to be true given $0 < d < 1$. Next, let $c < \hat{c}$ such that B sets the acceptance constraint. At $t = 2$, A and B both accept iff

$$x_2 p_B^l \geq p_C^m p_B^l(1-d)(1-c),$$

which defines

$$x_2 \geq p_C^m(1-d)(1-c) = x_2^B.$$

Once again, in the absence of shifting power, D is sure to propose $x_2^* = x_2^B$ and secure acceptance. Moving to the first period, B accepts iff

$$x_1 p_B^l+\delta x_2^B p_B^l \geq p_C^m p_B^h(1-d)(1-c)(1+\delta),$$

which defines

$$x_1 \geq \frac{p_C^m \left(p_B^h(1-c)(1-\delta) - \delta p_B^l\right)}{p_B^l} = x_1^A.$$

Unlike x_1^A, B's minimal demands can be greater than the per-period value of the pie, ensuring that it rejects all proposals $x \in [0,1]$, as represented by Line (4). Algebra shows that $x_1^B > 1$ is true when $c < 1 - p_B^l/p_B^h$,

$$d < 1 - \frac{p_B^l}{p_B^l\delta - p_B^h(1-c)(1+\delta)}, \quad \text{and} \quad p_C^h > \frac{p_B^l}{\left(p_B^h(1-c)(1-\delta) - p_B^l\delta\right)}.$$

Next, when $x_1^B \leq 1$, D prefers war to meeting the acceptance constraint at equality when

$$p_C^m(1-d)(1+\delta) > 1 - x_1^B + \delta(1-x_2^B),$$

also represented by Line (3), or when $c < 1 - p_B^l/p_B^h$,

$$d < 1 - \frac{p_B^l}{p_B^h(1-c)}, \quad \text{and} \quad p_C^h > \frac{dp_B^l}{(1-d)(p_B^h(1-c) - p_B^l)},$$

ensuring the existence of the SPE. □

Proof of Proposition 3. Two-player bargains are all based on the distribution of power and, by Lemma 2, A sets the acceptance constraint in this state. Therefore, I begin with A's choice over accepting or rejecting x_2, which B is also sure to accept if A does. A accepts iff

$$x_2(1-p_B^l) \geq p_C^l(1-p_B^l)(1-d),$$

which defines

$$x_2 \geq p_C^l(1-d) = x_2^A. \tag{11}$$

Once again, D is sure to propose $x_2^* = x_2^A$ and secure acceptance, given static power and the costs of war. Moving to the first period, A accepts iff

$$x_1(1-p_B^l) + \delta x_2^A(1-p_B^l) \geq p_C^m(1-p_B^l)(1-d)(1+\delta),$$

which defines

$$x_1 \geq \left(p_C^m + \delta(p_C^m - p_C^l)\right)(1-d) = x_1^A.$$

D will propose $x_1^* = x_1^A$ when it's feasible, but when $x_1^A > 1$, as re-expressed in Line (5), A rejects all proposals $x_1 \in [0,1]$. Algebra shows that $x_1^A > 1$ is satisfied when $\delta > (1-p_C^m)/(p_C^m - p_C^l)$,

$$d < 1 - \frac{1}{p_C^m(1+\delta) - p_C^l\delta}, \quad \text{and} \quad p_C^m > \frac{1+p_C^l}{2},$$

ensuring the existence of the proposed SPE. □

Proof of Proposition 4. Two-player bargains are based on the distribution of power, so I focus only on across-side offers x_t. First, let $c \geq \hat{c}$ such that A sets the acceptance constraint. At time $t = 2$, A accepts iff $x_2 \geq p_C^l(1-d) = x_2^A$ as defined in Line (11), and D is sure to set $x_2^* = x_2^A$ and induce acceptance. Moving to the first period, A accepts iff

$$x_1(1-p_B^l) + \delta x_2^A(1-p_B^l) \geq p_C^m(1-p_B^h)(1-d)(1+\delta),$$

which defines

$$x_1 \geq \frac{\left(p_C^m(1-p_B^h)(1+\delta) - p_C^l(1-p_B^l)\delta\right)(1-d)}{1-p_B^l} = x_1^A.$$

If $x_1^A \leq 1$, D meets the acceptance constraint at equality and induces acceptance. But when $x_1^A > 1$, as re-expressed in Line (6), A rejects any proposal $x_1 \in [0,1]$. The inequality $x_1^A > 1$ is true when δ is sufficiently large, $p_B^h > (1+p_B^l)/2$, $p_C^m > (1-p_B^l)/(2-2p_B^h)$,

$$p_C^l < \frac{2p_C^m(1-p_B^h)}{1-p_B^l} - 1, \quad \text{and} \quad d < 1 - \frac{1-p_B^l}{\left(p_C^m\left(1-p_B^h\right)(1+\delta) + p_C^l(1-p_B^l\right)\delta},$$

ensuring the existence of the SPE. Second, let $c < \hat{c}$ such that B sets the acceptance constraint. Beginning at $t = 2$, A sets the acceptance constraint, which as above dictates acceptance iff $x_2 \geq p_C^l(1-d) = x_2^A$ and which D is sure to meet at equality. Moving to the first period, where B sets the acceptance constraint, C accepts iff

$$x_1^A p_B^l + \delta x_2^A p_B^l \geq p_C^m p_B^h(1-c)(1-d)(1+\delta),$$

which defines

$$x_1 \geq \frac{\left(p_C^m p_B^h(1-c)(1+\delta) - p_C^l p_B^l \delta\right)(1-d)}{p_B^l} = x_1^B.$$

B's minimal demands can be greater than the per-period value of the pie, ensuring that it rejects all proposals $x \in [0,1]$, as represented by Line (8). Algebra shows that $x_1^B > 1$ is true when

$$d < 1 - \frac{p_B^l}{p_C^m p_B^h(1-c)(1+\delta) - p_C^l p_B^l \delta}, \quad p_B^h > \frac{1-c}{p_B^l}, \quad \text{and} \quad p_B^l < \frac{p_C^m}{p_B^h(1-c)}.$$

B's minimum demands can also fall outside the acceptance range for smaller different values of p_B^l and p_B^h, as long as δ is large enough, but as I'm interested only in existence, I present the simpler case here. Next, when $x_1^B \leq 1$, D prefers war to meeting the acceptance constraint at equality when

$$1 - x_1^B + \delta(1-x_2^A) \geq (1-p_C^m)(1-s)(1+\delta),$$

also represented by Line (7), or when

$$d < 1 - \frac{p_B^l}{p_B^h(1-c)}, \quad p_B^h > \frac{1-c}{p_B^l}, \quad \text{and} \quad p_B^l < \frac{p_B^h p_C^m(1-c)(1-d)}{d + p_C^m(1-d)},$$

ensuring the existence of the proposed SPE. □

Proof of Proposition 5. Let shifting intra-coalition power affect only p_C, such that $p_B = p_B^l = p_B^h$. If intra-coalition power is static, C wins a continued war with probability p_C^m; if C is rising, it wins with probability p_C^h; and if it's declining, it wins with probability p_C^l, where $p_C^l < p_C^m < p_C^h$. We know from Lemma 2 that A sets the acceptance constraint in both periods, so the terms of the second period bargain—which given the absence of shifting power is sure to be accepted—is $x_2^A = p_C^{m,l}(1-d)$, where the superscript is m in cases $s(0,B)$ and l in cases $s(1,B)$. Moving back to the first period, let $s = 1$ indicate that fighting alters the coalition's chances of winning, whether raising or lowering it, and $s = 0$ indicate that fighting doesn't alter p_C. A accepts iff

$$x_1(1-p_B)+\delta x_2^A(1-p_B) \geq \left(sp_C^{h,l}+(1-s)p_C^m\right)(1-p_B)(1-d)(1+\delta).$$

The required offer is outside the feasible range, such that $x_1^C > 1$ when

$$\left(sp_C^{h,l}+(1-s)p_C^m\right)-\delta p_C^{m,l} > \frac{1}{1-d},$$

where the left side is easier to satisfy if $s = 1$ and $p_C = p_C^h$, relative to the baseline of $s = 0$, and easier to satisfy if $s = 1$ and $p_C = p_C^l$. □

Extension: Shifting Coalition Power

This section considers two cases in which shifting power between coalition partners affects the across-side distribution of power. Formally, I let $p_B^l = p_B^h$ to abstract away from changing within-side bargains, which lets me isolate two possible effects on across-side power. First, as discussed above, B's capabilities may increase, which increases the coalition's share of across-side power to p_C^h if the war continues. Second, it may also be the case that A's capabilities will shrink if the war continues, giving B the advantage it enjoys in the main analysis but causing power to shift ever further against the coalition. Therefore, fighting doesn't merely prevent D's power from increasing; it can also actively increase or decrease the coalition's total power, and therefore its expected share of the pie if it continues the war in the first period. Since p_B is fixed, we know from Lemma 2 that A sets the acceptance constraint. Fixed within-side power also ensures that the coalition is a *de facto* unitary belligerent, so these results apply equally well to two-player models of war termination where fighting can actively change a player's capabilities—for example, if it's yet to fully mobilize its population for the war effort when the game begins or if continuing the war will undermine the domestic consensus in favor of war.

Proposition 5. *When shifting power inside the coalition affects p_C only, wars are weakly longer when fighting raises p_C and weakly shorter when fighting lowers p_C.*

Proposition 5 states two results. First, when fighting weakens the coalition—say, if its own relative power will approach p_C^l over the course of fighting—then it's more likely to end the war in the first period, tolerating D's rise after a pause rather than pay to let its capabilities deteriorate through fighting. Second, when fighting will strengthen the coalition, as it would in the main analysis where B's capabilities increase, C is less likely to end the war. We can summarize the result with the inequality,

$$\left(sp_C^h + (1-s)p_C^m\right)(1+\delta) - p_C^{m,l}\delta > \frac{1}{1-d}$$

where $s = 1$ when power is shifting in the coalition's favor, $s = 0$ when it isn't, and $p_C^{m,l}$ represents the coalition's relative power at $t = 1$ if the war ends, p_C^m when D isn't rising and p_C^l when D is rising. For any second-period probability of defeating D—i.e., whether or not power is shifting across sides—the total shift in power increases in p_C^h. This ensures that when $s = 1$ the coalition keeps fighting under conditions that would otherwise see the war end when $s = 0$, whether or not the underlying bargaining problem has been solved.

This result identifies a commitment problem unappreciated in models of war termination. Country D may be unable to compensate C for a shift in power that occurs if the war ends, but D is also unable to compensate the coalition for the position of relative power the latter expects to occupy if the war continues. The end of the Gulf War of 1991 offers a useful example. After demonstrating that it could eject Iraqi forces from Kuwait at acceptable speed and cost, resolving the uncertainty that Iraq to resist an ultimatum to withdraw (Lindsey 2015, 636-637), the American-led coalition continued to pursue retreating Iraqi armor on the so-called "Highway of Death," effecting a shift in the regional distribution of power that would've been impracticable with a war that ended as soon as Iraqi forces were on the run. In February 1991, the difference was a matter of hours, but in principle a coalition that grows stronger in the fighting could extend the war for as long as it might take to realize such a shift in power. In contrast to private benefits gained from fighting (Powell 2017, Spaniel, Bils and Judd 2020), this result shows that cumulative gains in relative power produced by fighting, sometimes related to the offense-defense balance (Van Evera 1998, 8), represent a commitment problem; the war could end, if only D could compensate its rising enemy for the increased strength it would forego by ending the war.