Post-Shift Histories

We begin by characterizing behavior in post-shift continuations, which follows one of four histories, $h_1-h_4$. These histories describe the state of $A$’s information about $B$’s type, the number of previous battles, and whether a shift in power has occurred. In $h_1$, there has been one previous battle and $A$ knows $B$’s type; in $h_2$, there has been one previous battle and $A$ does not know $B$’s type; in $h_3$, there have been no previous battles and $A$ knows $B$’s type; and in $h_4$, there have been no previous battles and $A$ does not know $B$’s type. Histories place no restrictions on the timing or ordering of previous battles and belief updating. We focus on stationary Perfect Bayesian strategies in which players make the same choices whenever they have identical beliefs and the rest of the game is structurally identical. Finally, recall our restriction over the size of per-battle costs to ensure that there are no corner proposals, e.g. $x_n = 1$, in post-shift histories. We do this to highlight the fact that the shift in power can create corner proposals in pre-shift histories, as discussed below.

History $h_1$

At history $h_1$, $A$ knows $B$’s type, so its beliefs are either $\beta = 0$ or $\beta = 1$. Attack or rejection ends the game in a decisive battle. If $B$ accepts a proposal $x_{1T}$, then stationarity implies that $A$ can
proposes $x_{1T}$ in all future periods, and $B$ will accept it. A player of type $b_T$ accepts some $x_{1T}$ rather than reject iff
\[
\frac{1 - x_{1T}}{1 - \delta} \geq \left(1 - \frac{p}{1 + s}\right) - \frac{1}{1 - \delta} - b_T \Leftrightarrow x_{1T} \leq \frac{p}{1 + s} + (1 - \delta)b_T.
\] (1)

Thus, $A$'s most preferred proposal that is also acceptable to type $b_T$ is $x_{1T}^* = p/(1 + s) + (1 - \delta)b_T$, because it is the minimum required to win acceptance. Proposing more provokes rejection, while proposing less wins acceptance of a less attractive proposal. (By this logic, $A$ always proposes to keep as much as possible when it sets equilibrium proposals, so we only mention it in detail here.) $A$ proposes $x_{1T}^*$, rather than attack iff
\[
x_{1T}^* \geq \frac{p}{1 + s} - a \Leftrightarrow a + b_T > 0,
\]
which is true by assumption. Therefore, $A$'s equilibrium proposal at history $h_1$ is $x_{1T}^* = p/(1 + s) + (1 - \delta)b_T$, which $B$ accepts.

**History $h_2$**

At history $h_2$, $A$ has not yet updated its beliefs, so $\beta = q$. As with $h_1$, attack or rejection ends the game in a decisive battle. This ensures that player-type $b_T$'s payoffs to rejection are the same as in history $h_1$. $A$'s available actions are to attack, which ends the game; to make a pooling proposal, which both types accept and keeps the game in history $h_2$; and to make a separating proposal, which the weak accepts and the strong rejects, and which moves the game to history $h_1$ upon acceptance.

By the logic of history $h_1$, $A$'s optimal pooling proposal will be $y_{2L}^* = p/(1 + s) + (1 - \delta)b_L$. Since the strong type accepts $x_{2L}^*$, so must the weak type, since its payoff to rejection is lower than the strong type's, i.e. $b_H > b_L$. Further, Inequality (1), also ensures that $A$ prefers $x_{2L}^*$ to attack. Thus, forcing a battle is never sequentially rational at history $h_2$. For a separating proposal, the weak type accepts and the strong rejects, and the incentive
compatibility constraints are

\[
1 - x_2^H + \delta \frac{1 - x_1^H}{1 - \delta} \geq \left(1 - \frac{p}{1 + s}\right) \frac{1}{1 - \delta} - b_H \quad (b_H)
\]

\[
\left(1 - \frac{p}{1 + s}\right) \frac{1}{1 - \delta} - b_L > 1 - x_2^H + \delta \frac{1 - x_1^H}{1 - \delta} \quad (b_L),
\]

which reduce to

\[
1 + \delta \frac{1 - x_1^H}{1 - \delta} - \left(1 - \frac{p}{1 + s}\right) \frac{1}{1 - \delta} + b_L < x_2^H \leq 1 + \delta \frac{1 - x_1^H}{1 - \delta} - \left(1 - \frac{p}{1 + s}\right) \frac{1}{1 - \delta} + b_H.
\]

A prefers to make the largest possible proposal satisfying these constraints, so its optimal separating proposal, after substitution, is

\[
x_1^* \equiv \frac{p}{1 + s} + \frac{b_H - b_L}{a + b_H} = \bar{\beta},
\]

and it makes a separating proposal when \(\beta < \bar{\beta}\). Conversely, A prefers a separating proposal to a one-period deviation to a pooling proposal when \(U_A(x_2^H) > x_2^* + \delta U_A(x_3^H)\), or when \(\beta < (b_H - b_L)/(a + b_H) = \bar{\beta}\). Therefore, at history \(h_2\), A proposes \(x_2^H\) (accepted by \(b_H\) and rejected by \(b_L\)) when \(\beta < \bar{\beta}\) and \(x_2^L\) (accepted by \(b_T\)) when \(\beta \geq \bar{\beta}\).

**History \(h_3\)**

At history \(h_3\), no battles have occurred, and A knows B’s type, so its beliefs are \(\beta = 0\) or \(\beta = 1\). Acceptance of a proposal keeps the game in history \(h_3\), while rejection or attack transitions the game to history \(h_1\). If B accepts a proposal \(x_3^T\), then stationarity implies that A can propose \(x_3^T\) in all future periods, and B will accept it. However, since rejection does not end the game at histories with no previous battles, B’s rejection of any proposal that it should accept is out of equilibrium, and we stipulate that A believes \(\beta' = 1\) in the event of any unexpected rejection. In
other words, by rejecting a proposal, player-type $b_T$ convinces $A$ that it faces the strong type, and $A$ behaves accordingly at history $h_1$. Therefore, player-type $b_T$ will accept some $x_{3T}$ iff

$$\frac{1 - x_{3T}}{1 - \delta} \geq -b_T + \delta \frac{1 - x^*_L}{1 - \delta} \iff x_{3T} \leq 1 + (1 - \delta) b_T - \delta (1 - x^*_L).$$

$A$’s most preferred proposal that is acceptable to player-type $b_T$ is $x^*_{3T} = 1 + (1 - \delta) b_T - \delta (1 - x^*_L)$, and $A$ makes such a proposal in each period rather than attack iff

$$\frac{x^*_{3T}}{1 - \delta} \geq -a + \delta \frac{x^*_L}{1 - \delta} \iff a + b_T \geq 0,$$

which is true by assumption. Likewise, $A$ strictly prefers a one-period deviation to $x^*_{3T}$ over attacking in every period iff

$$x^*_{3T} + \delta (-a) + \delta^2 \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - a \right) \geq -a + \delta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - a \right) \iff a + b_T \geq 0,$$

which is true by assumption. Therefore, $A$’s equilibrium proposal at history $h_3$ is $x^*_{3T} = 1 + (1 - \delta) b_T - \delta (1 - x^*_L)$, which type $b_T$ accepts.

**History $h_4$**

At history $h_4$, no battles have occurred, and $A$ is uncertain over which type it faces, so $\beta = q$. If $A$ attacks, the game transitions to history $h_2$. If $A$ makes a separating proposal, the game transitions to history $h_1$ upon rejection and history $h_3$ upon acceptance. If it makes a pooling proposal, the game remains in history $h_4$. Recall that $A$ responds to any out-of-equilibrium rejection by believing $\beta' = 1$, which transitions the game to history $h_1$.

$A$’s optimal pooling proposal will be the largest proposal that ensures the strong type’s acceptance, and this will also ensure the weak type’s acceptance. To verify this, note that the strong type accepts a proposal $x_{4L}$ in every period iff

$$\frac{1 - x_{4L}}{1 - \delta} \geq -b_L + \delta \frac{1 - x^*_L}{1 - \delta} \iff x_{4L} \leq 1 + (1 - \delta) b_L - \delta (1 - x^*_L).$$

The weak type, on the other hand, receives $-b_H + \delta (1 - x^*_L)/(1 - \delta)$ from rejection, since its out-of-equilibrium rejection convinces $A$ that $\beta' = 1$, which is strictly less than the strong type’s
payoff to rejection. Therefore, player-type $b_H$ accepts all proposals acceptable to $b_L$, and $A$ sets its pooling proposal at $x_{4L}^* = 1 + (1 - \delta)b_L - \delta (1 - x_{1L}^*)$, which both types of $B$ accept.

A separating proposal is accepted by the weak type but rejected by the strong. The weak type must prefer not to mimic the strong by rejecting and receiving more generous deals in the future, while the strong type must prefer to reject rather than mimic the weak type. To define the strong type's payoff for accepting, note that should player-type $b_L$ deviate from the strategy of rejection by accepting a separating proposal, $A$ believes $\beta = 0$ and proposes $x_{3H}^*$ in the next period, which player-type $b_L$, returning to optimal play, rejects. This engages $A$'s out-of-equilibrium beliefs $\beta' = 1$ for the next period, in which it proposes $x_{1L}^*$, which by stationarity $B$ accepts in every subsequent period. Therefore, the incentive compatibility constraints are

$$1 - x_{4H} + \delta \frac{1 - x_{3H}^*}{1 - \delta} \geq -b_H + \delta \frac{1 - x_{1L}^*}{1 - \delta}, \quad (b_H)$$

$$-b_L + \delta \frac{1 - x_{1L}^*}{1 - \delta} > 1 - x_{4H} + \delta (-b_L) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta}, \quad (b_L),$$

which reduce to

$$1 - \delta (1 - x_{1L}^*) + (1 - \delta) b_L < x_{4H} \leq 1 - \frac{\delta}{1 - \delta} (x_{3H}^* - x_{1L}^*) + b_H.$$ 

This inequality holds as long as $b_H > b_L$ and $0 < \delta < 1$, so $A$'s optimal separating proposal is $x_{4H}^* = 1 - \delta (x_{3H}^* - x_{1L}^*)/(1 - \delta) + b_H$, which the strong type rejects and the weak accepts.

Now consider $A$'s strategy. Its payoff to attacking at history $h_4$, which transitions the game to history $h_2$, depends on the value of $\beta$. Recall that, at history $h_2$, $A$ makes a separating proposal when $\beta < \bar{\beta}$ and a pooling proposal when $\beta \geq \bar{\beta}$. Our first step is to show that, for all values of $\beta$, $A$ will not attack. This allows for a subsequent comparison between separating and pooling proposals. Begin with $\beta < \bar{\beta}$, where $A$'s payoff to attack is

$$U_A(\text{att}_4 | \beta < \bar{\beta}) = -a + \delta \left[ \beta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - a \right) + (1 - \beta) \left( x_{2H}^* + \delta \frac{x_{1H}^*}{1 - \delta} \right) \right],$$

which is less attractive than a separating proposal, $x_{4H}^*$, iff

$$\beta \left( -a + \delta \frac{x_{1L}^*}{1 - \delta} \right) + (1 - \beta) \left( x_{4H}^* + \delta \frac{x_{3H}^*}{1 - \delta} \right) > U_A(\text{att}_4 | \beta < \bar{\beta}) \iff \beta < \frac{b_H - b_L}{1 + b_H} = \bar{\beta}.$$
Therefore, when $\beta < \bar{\beta}$, making a separating proposal dominates attack. Next, when $\beta \geq \bar{\beta}$, $A$'s payoff to attack is $U_A(\text{att}|\beta \geq \bar{\beta}) = -a + \delta x_{2L}^* / (1 - \delta)$, which is less attractive than a separating proposal iff

$$\beta \left(-a + \delta \frac{x_{1L}^*}{1 - \delta}\right) + (1 - \beta) \left(x_{4H}^* + \delta \frac{x_{3H}^*}{1 - \delta}\right) > -a + \delta \frac{x_{2L}^*}{1 - \delta},$$

or when $\beta < 1$, $b_H > b_L > 0$, and $a > 0$. Therefore, $A$ strictly prefers separating proposals to attacking at history $h_4$ for all values of $\beta$.

Finally, $A$ prefers the pooling proposal in each period to the separating proposal iff

$$\frac{x_{4L}^*}{1 - \delta} \geq \beta \left(-a + \delta \frac{x_{1L}^*}{1 - \delta}\right) + (1 - \beta) \left(x_{4H}^* + \delta \frac{x_{3H}^*}{1 - \delta}\right) \iff \beta \geq \frac{b_H - b_L}{1 + a + b_H} \equiv \underline{\beta},$$

and prefers the separating proposal iff $\beta < \underline{\beta}$. Likewise, $A$ prefers making the separating proposal to a one-period deviation to the pooling proposal iff $U_A(x_{4H}^*) > x_{4L}^* + \delta U_A(x_{4H}^*)$, or when $\beta < (b_H - b_L) / (1 + a + b_H) = \bar{\beta}$. Therefore, in each period where the history is $h_4$, $A$ proposes $x_{4H}^*$ when $\beta < \underline{\beta}$ and $x_{4L}^*$ when $\beta \geq \underline{\beta}$, type $b_L$ accepts $x_{4L} \leq x_{4L}^*$, and type $b_H$ accepts $x_{4H} \leq x_{4H}^*$.

Before considering pre-shift histories, it is worth noting that $\underline{\beta} < \bar{\beta}$, or that $A$ is generally more willing to make separating proposals after one battle (history $h_2$) than after no battles ($h_4$). The discussion of equilibrium strategies given the four possible post-shift histories implies that, first, when $A$ faces only one type of $B$ after a shift in power, it makes an acceptable proposal to that type of $B$ in all subsequent periods. Second, when $A$ remains uncertain over which type it faces, there are three cases to consider: (a) when $\beta \geq \bar{\beta}$, $A$ makes a pooling proposal that both types accept in each period; (b) when $\underline{\beta} \leq \beta < \underline{\beta}$, $A$ makes pooling proposals in each period if there have been no battles but makes a separating proposal if there has been one battle; and (c) when $\beta < \underline{\beta}$, $A$ makes separating proposals.

**Pre-Shift Histories**

We now move back up the game tree to consider pre-shift histories, $h_5$-$h_7$, and we characterize equilibrium behavior in the following format. For each history, we begin with a definition
of equilibrium proposals that identify, first, the pooling proposal acceptable to both types of
B and, second, the separating proposal acceptable to the weak type but unacceptable to the
strong. Then, for each history we identify the conditions under which A will choose each of the
options available to it—the pooling proposal, the separating proposal, and attack. Note that
in pre-shift play, attack can be sequentially rational. Because A’s share of the benefits in any
given round cannot exceed one, for some values of s, A cannot receive enough of the benefits
in the current period to prevent it from acting to forestall the shift. For each of the subsequent
histories we identify when this constraint on x binds as a function of s, i.e. when the shift in
power will be so large that there exists no proposal that can induce A not to forestall the shift by
attacking. Given anticipated sequentially rational behavior and beliefs established above, this
will complete the characterization of PBE for each part of the parameter space. We begin with
histories at which one battle has been fought, h₅ and h₆, then consider history h₇, at which A’s
first move occurs.

**History h₅**

At history h₅, one battle has been fought, and A knows B’s type. Rejection or attack ends the
game in a decisive battle and prevents a shift in power from occurring, while acceptance leads
to a shift in power and history h₁, where the next battle is decisive. B accepts pre-shift proposals
x₅T that satisfy

\[ 1 - x₅T + \delta \frac{1 - x^*_T}{1 - \delta} \geq \frac{1 - p}{1 - \delta} - b_T \iff x₅T \leq 1 - \frac{1}{1 - \delta} (1 - p - \delta (1 - x^*_T)) + b_T. \]  

(2)

A’s possible replies are to attack or propose x₅T*, allowing a shift in power and engaging
strategies that follow history h₁. However, B cannot commit not to use its increased bargaining
power after a shift, and as such it has incentives to be very generous to A before a shift in power
(especially as \( \delta \rightarrow 1 \)). The most it can promise to A in the present is \( x₅T = 1 \), and A chooses to
attack rather than make such a proposal when
\[
\frac{p}{1 - \delta} - a > 1 + \delta \frac{x^*_{1T}}{1 - \delta} \iff s > \frac{p \delta}{p + \delta - (1 - \delta) (a + \delta b_T) - 1} - 1.
\] (3)

Otherwise, there is sure to exist some proposal satisfying Inequality (2) that \( A \) prefers to make rather than attack. To see this, note that if \( A \) proposes some \( x^T_5 \) that satisfies (2) at equality, then \( x^T_5 + \delta x^*_1 T / (1 - \delta) \geq p / (1 - \delta) - a \) is strictly true, but it may not the the case that \( x^T_5 \leq 1 \). But when \( s \) is small enough that (3) does not hold, \( A \)'s equilibrium proposal \( x^*_5 \) is therefore the minimum of the constraint in Inequality (2) and 1. Formally, its equilibrium proposal is
\[
x^*_5 = \min \{ 1 - \frac{1}{1 - \delta} (1 - p - \delta (1 - x^*_1 T)) + b_T, 1 \}.
\]

More generally for all pre-shift proposals, \( s \) can be so large that equilibrium proposals can have corner solutions, i.e. \( x_n = 1 \), but rather than re-derive it for each proposal, we simply note in the remainder of the proof that \( A \) will either meet the proper constraint at equality or set it at 1 when \( s \) is sufficiently large.

Finally, Inequality (3) shows that \( A \)'s choice over attack or making an acceptable proposal depends on the player-type it believes that it faces at this information set, \( b_T \). When \( \beta = 1 \), \( A \) attacks iff
\[
s > \frac{p \delta}{p + \delta - (1 - \delta) (a + \delta b_T) - 1} - 1 \equiv \hat{s}
\] (4)
and otherwise proposes \( x^*_5 \), which the strong type accepts. When \( \beta = 0 \), \( A \) attacks iff
\[
s > \frac{p \delta}{p + \delta - (1 - \delta) (a + \delta b_H) - 1} - 1 \equiv \hat{s}
\] (5)
and otherwise proposes \( x^*_5 \), which the weak type accepts.

**History \( h_6 \)**

At history \( h_6 \), there has been one battle, and \( A \) does not know which type of \( B \) it faces. Rejection or attack again ends the game in a decisive battle, and acceptance engages either history \( h_1 \) (\( A \)
knows B’s type) or $h_2$ (A does not know B’s type), depending on whether A makes a separating or pooling proposal. Should B accept a proposal, the consequences depend on its type and on the response of the other type to the same proposal (i.e. separating or pooling).

To define the pooling proposal, we begin with the strong type’s acceptance rule, which is conditional on the value of $\beta$, since the weak will also accept and engage history $h_2$. However, regardless of whether A makes a separating or pooling proposal after history $h_2$, player-type $b_L$ receives an equivalent payoff—a decisive battle in response to a separating proposal or its certainty equivalent, $x_{2L}^*$, in a pooling proposal—and therefore we can define a single proposal required to win the strong type’s acceptance for any value of $\beta$. Using $x_{2L}^*$ to simplify the presentation, the strong type accepts proposals satisfying

$$1 - x_{6L} + \delta \frac{1 - x_{2L}^*}{1 - \delta} \geq (1 - p) \frac{1}{1 - \delta} - b_L \Leftrightarrow x_{6L} \leq 1 - \frac{1}{1 - \delta} (1 - p - \delta(1 - x_{2L}^*)) + b_L.$$ 

The weak type accepts such a proposal when

$$1 - x_{6L} + \delta \frac{1 - x_{2L}^*}{1 - \delta} \geq (1 - p) \frac{1}{1 - \delta} - b_H,$$

where its payoff differs from the strong type’s in that it pays greater per-battle costs, $b_H > b_L$. Therefore, since player-type $b_H$ is worse off by rejecting, it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal, then, A sets

$$x_{6L}^* = \min\{1 - \frac{1}{1 - \delta} (1 - p - \delta(1 - x_{2L}^*) + b_L), 1\}.$$ 

Separating proposals induce the weak type’s acceptance and the strong type’s rejection. Acceptance transitions the game into history $h_1$, at which A believes $\beta = 0$ and proposes $x_{1H}^*$, which $b_H$ accepts, in all post-shift periods. The incentive compatibility constraints are

$$1 - x_{6H} + \delta \frac{1 - x_{1H}^*}{1 - \delta} \geq p \times \frac{1}{1 - \delta} - b_H \quad (b_H)$$

$$p \times \frac{1}{1 - \delta} - b_L > 1 - x_{6H} + \delta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - b_L \right) \quad (b_L),$$

which reduce to

$$1 + \delta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - b_L \right) - \frac{p}{1 - \delta} + b_L < x_{6H} \leq 1 - \frac{1}{1 - \delta} (1 - p - \delta(1 - x_{1H}^*)) + b_H \quad (6)$$

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Inequality (6) holds as long as \( b_H > b_L \), ensuring that \( A \) can set proposals to induce rejection by the strong and acceptance by the weak. Therefore, to make a separating proposal, \( A \) sets
\[
x_{6H}^* = \min\{1 - \frac{1}{1 - \delta}(1 - p - \delta(1 - x_{1H}^*)) + b_H, 1\}.
\]

\( A \)'s has three choices: attack, a separating proposal, and a pooling proposal. While \( A \)'s payoffs to attacking or making a separating proposal do not depend on \( \beta \), its payoff to making a pooling proposal does, since doing so moves the game into history \( h_2 \). Thus, we first characterize \( A \)'s best reply when \( \beta \geq \bar{\beta} \) and follow by discussing \( \beta < \bar{\beta} \).

When \( \beta \geq \bar{\beta} \), \( A \)'s payoff from making a separating proposal is
\[
U_A(x_{6H}^*) = \beta \left( \frac{p}{1-\delta} - a \right) + (1 - \beta) \left( x_{6H}^* + \delta \frac{x_{1H}^*}{1-\delta} \right),
\]
where its post-shift per-period payoff of \( x_{1H}^* \) is derived from history \( h_1 \). \( A \)'s payoff for a pooling proposal is
\[
U_A(x_{6L}^*) = x_{6L}^* + \delta x_{2L}^*/(1 - \delta),
\]
where its post-shift per-period payoff of \( x_{2L}^* \) is derived from history \( h_2 \). However, \( A \) prefers attacking to the most generous possible pooling proposal, \( x_{6L} = 1 \), today iff
\[
\frac{p}{1-\delta} - a > 1 + \delta \frac{x_{2L}^*}{1-\delta} \Leftrightarrow s > \underline{s},
\]
where \( \underline{s} \) is the same constraint as \( s \) defined above. Further, \( A \) prefers attack to the constrained maximum separating proposal, \( x_{6H} = 1 \), iff
\[
\frac{p}{1-\delta} - a > \beta \left( \frac{p}{1-\delta} - a \right) + (1 - \beta) \left( 1 + \delta \frac{x_{1H}^*}{1-\delta} \right) \Leftrightarrow s > \underline{\delta},
\]
as defined above. Note that, since \( b_L < b_H \), it is also the case that \( \underline{\delta} > \underline{s} \). Finally, when \( A \) does not prefer war to the most generous pooling proposal, or \( s \leq \underline{s} \), we compare the separating and pooling proposals, and algebra shows that \( A \) never prefers the optimal separating proposal, \( x_{6H}^* \), as long as \( \beta > (b_H - b_L)/(a + b_H) = \bar{\beta} \), which is the constraint that defines this subcase. Therefore, at history \( h_6 \) when \( \beta \geq \bar{\beta} \), \( A \)'s best responses are summarized as follows. \( A \) proposes \( x_{6L}^* \) when \( s \leq \underline{s} \), \( x_{6H}^* \) when \( \underline{s} < s \leq \underline{\delta} \) and attacks when \( s > \underline{\delta} \).
When $\beta < \bar{\beta}$, $A$’s payoffs for attack and separating proposals remain unchanged, and its payoff for making a pooling proposal and entering history $h_2$ is $U_A(x_{6L}^*)$, or

$$U_A(x_{6L}^*) = x_{6L}^* + \delta \left[ \beta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - a \right) + (1 - \beta) \left( x_{2H}^* + \delta \frac{x_{1H}^*}{1 - \delta} \right) \right].$$

However, no such proposal—not even $x_{6L} = 1$ can induce $A$ not to attack iff

$$\frac{p}{1 - \delta} - a > 1 + \delta \left[ \beta \left( \frac{p}{1 + s} \times \frac{1}{1 - \delta} - a \right) + (1 - \beta) \left( x_{2H}^* + \delta \frac{x_{1H}^*}{1 - \delta} \right) \right],$$

or when

$$s > -\frac{(-1 + \delta)(-1 + p + a(-1 + \beta\delta) + (-1 + \beta)\delta b_H)}{1 - p - \delta + a(-1 + \delta)(-1 + \beta\delta) + (-1 + \beta)(-1 + \delta)\delta b_H}. \tag{7}$$

Algebra shows that the constraint in Inequality (7) is less than $\hat{s}$, and the pooling proposal exists only when (7) does not hold. As in the previous subcase, there exists a separating proposal only when $s < \hat{s}$, and $A$ prefers attack when $s > \hat{s}$. However, when $A$ does not prefer war to the most generous pooling proposal, i.e. when (7) does not hold, it strictly prefers to make the separating proposal iff $U_A(x_{6H}^*) > U_A(x_{6L}^*)$, or $\beta < (b_H - b_L)/(a + b_H) = \bar{\beta}$, which is the constraint that defines the case. When $s \leq s < \hat{s}$ $A$ prefers attack to the pooling proposal but not to the separating. Therefore, at history $h_6$ when $\beta < \bar{\beta}$, $A$ attacks when $s > \hat{s}$ and makes the separating proposal $x_{6H}^*$ when $s \leq \hat{s}$.

To summarize results thus far, taking into account $A$’s equilibrium behavior from histories $h_5$ and $h_6$, the equilibrium space is two-dimensional. First, when $\beta = 1$ ($h_5$), $A$ attacks when $s > \hat{s}$ and proposes $x_{5L}^*$ in order to transition to $h_1$ when $s \leq \hat{s}$. Second, when $\beta = 0$ ($h_5$), $A$ attacks when $s > \hat{s}$ and proposes $x_{5H}^*$ in order to transition to $h_1$ when $s \leq \hat{s}$. Third, when $\beta \geq \bar{\beta}$ at history $h_6$, $A$ proposes $x_{6L}^*$ in order to transition to history $h_2$ when $s \leq \hat{s}$, proposes $x_{6H}^*$ in order to transition to history $h_1$ when $s < \hat{s}$; and attacks when $s > \hat{s}$. Fourth, when $\beta < \bar{\beta}$ at history $h_6$, $A$ attacks when $s > \hat{s}$ and proposes $x_{6H}^*$ in order to transition into history $h_1$ upon acceptance when $s \leq \hat{s}$. Finally, it is worth noting that, for $b_H$, $b_L$, and $a$ sufficiently low, it is also the case that $\hat{s} > \hat{s}$. We maintain these restrictions through the remainder of the proofs.
**History** $h_7$

We now consider history $h_7$, which corresponds to $A$’s initial decision node and $B$’s first response to a proposal. At this history no battles have been fought, and $A$ is uncertain over which type it faces, beginning the game with prior beliefs $\beta = q$. The analysis of histories $h_1-h_6$ imply that, with $A$’s initial choices of attacking, making a separating proposal, and making a pooling proposal, there are nine combinations of parameter values to consider that imply unique sets of choices and outcomes down the path of play. Below, when we say that $A$ will “satisfy the strongest type present,” we mean that it issues a pooling proposal if it faces both types and a proposal acceptable to whatever type is present if it faces only one. The nine cases are:

1. $\beta \geq \bar{\beta}$ and $s \leq \bar{s}$. *Post-battle, pre-shift:* satisfy the strongest type present. *Post-shift:* satisfy the strongest type present.

2. $\beta \geq \bar{\beta}$ and $\bar{s} < s \leq \bar{s}$. *Post-battle, pre-shift:* if facing both types, make a separating proposal; if one type, attack the strong but satisfy the weak. *Post-shift:* satisfy the strongest type present.

3. $\beta \geq \bar{\beta}$ and $s > \bar{s}$. *Post-battle, re-shift:* attack regardless of beliefs. *Post-shift:* satisfy the strongest type present.

4. $\underline{\beta} \leq \beta < \bar{\beta}$ and $s \leq \bar{s}$. *Post-battle, pre-shift:* if facing both types, make a separating proposal; if one type, satisfy it. *Post-shift:* if both types, make a separating after one battle but a pooling proposal after no battles; if one type, satisfy it.

5. $\underline{\beta} \leq \beta < \bar{\beta}$ and $\bar{s} < s \leq \bar{s}$. *Post-battle, pre-shift:* if both types, make a separating proposal; if one type, attack the strong but satisfy the weak. *Post-shift:* if both types, make a separating after one battle but a pooling proposal after no battles; if one type, satisfy it.

6. $\underline{\beta} \leq \beta < \bar{\beta}$ and $s > \bar{s}$. *Post-battle, pre-shift:* attack regardless of beliefs. *Post-shift:* if both types, make a separating after one battle but a pooling proposal after no battles; if one
type, satisfy it.

7. $\beta < \underline{\beta}$ and $s \leq \hat{s}$. 
   \textit{Post-battle, pre-shift}: if both types, make a separating proposal; if one type, satisfy it. 
   \textit{Post-shift}: if both types, make a separating proposal; if one type, satisfy it.

8. $\beta < \underline{\beta}$ and $\underline{s} < s \leq \hat{s}$. 
   \textit{Post-battle, pre-shift}: if both types, make a separating proposal; if one type, attack the strong but satisfy the weak. 
   \textit{Post-shift}: if both types, make a separating proposal; if one type, satisfy it.

9. $\beta < \underline{\beta}$ and $s > \hat{s}$. 
   \textit{Post-battle, pre-shift}: attack regardless of beliefs. 
   \textit{Post-shift}: if both types, make a separating proposal; if one type, satisfy it.

As above, we specify each player-type’s best response rules as a function of anticipated strategies and beliefs (which we showed to be consistent and sequentially rational above), which are conditional on the parameter values of $\beta$ and $s$ that define each of the nine cases. We begin with each type of $B$’s acceptance constraints and then examine $A$’s choice of attack, separating proposal, and pooling proposal in turn, following the format established in histories $h_5$ and $h_6$.

\textbf{Case 1. $\beta \geq \underline{\beta}$ and $s \leq \hat{s}$.}

\textit{Post-battle, pre-shift}: satisfy the strongest type present. \textit{Post-shift}: satisfy the strongest type present.

First, define a pooling proposal as one that both types of $B$ accept, which implies that any type’s rejection is out-of-equilibrium. This engages $A$’s out-of-equilibrium beliefs, $\beta' = 1$. The strong type accepts when

$$1 - x_{7L} + \delta \frac{1 - x_{4L}^*}{1 - \delta} \geq -b_L + \delta (1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta},$$

or when

$$x_{7L} \leq 1 - \delta \left( (1 - x_{5L}^*) + \delta \frac{1 - x_{1L}^*}{1 - \delta} - \frac{1 - x_{4L}^*}{1 - \delta} \right) + b_L.$$
The weak type accepts such a proposal when
\[ 1 - x_7 + \delta \frac{1 - x_{5L}^*}{1 - \delta} \geq -b_H + \delta(1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta}, \]
where its payoffs differ only from the strong type’s in that it must may a higher per-battle cost if it rejects a proposal, \( b_H > b_L \), to go on to accept proposals consistent with \( A \)’s out-of-equilibrium beliefs \( \beta' = 1 \). Therefore, since the weak is worse off by rejecting, it is sure to accept all proposals also acceptable to the strong type, and to make a pooling proposal that both types accept, \( A \) sets
\[ x_{7L}^* = \min\{1 - \delta \left( (1 - x_{5L}^*) + \delta \frac{1 - x_{1L}^*}{1 - \delta} - \frac{1 - x_{3L}^*}{1 - \delta} \right) + b_L, 1\}. \]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. First, the strong type must prefer to reject, honestly revealing its type, over accepting and mimicking the weak type; this results in the strong type rejecting \( A \)’s subsequent proposals upon its return to optimal behavior, engaging \( A \)’s out-of-equilibrium beliefs \( \beta' = 1 \) and finally guaranteeing itself a payoff based on \( x_{1L}^* \). The incentive compatibility constraints are
\[ 1 - x_{7H} + \delta \frac{1 - x_{3H}^*}{1 - \delta} \geq -b_H + \delta(1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} \quad (b_H) \]
\[ -b_L + \delta(1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} > 1 - x_{7H} + \delta(-b_L) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} \quad (b_L), \]
which reduce to
\[ 1 + (1 - \delta)b_L - \delta(1 - x_{5L}^*) < x_{7H} \leq 1 - \delta \left( (1 - x_{5L}^*) + \delta \frac{1 - x_{1L}^*}{1 - \delta} - \frac{1 - x_{3H}^*}{1 - \delta} \right) + b_H. \]
This inequality holds, ensuring the existence of a separating proposal, when \( b_H > b_L \), which is true by construction. Therefore, \( A \)’s optimal separating proposal is
\[ x_{7H}^* = \min\{1 - \delta \left( (1 - x_{5L}^*) + \delta \frac{1 - x_{1L}^*}{1 - \delta} - \frac{1 - x_{3H}^*}{1 - \delta} \right) + b_H, 1\}. \]

To characterize \( A \)’s best reply, we follow the logic used above to identify when, as a function of \( s \), there exist proposals that \( A \) prefers to attack. For this case, we first show that, when \( A \) does
not prefer war to the most generous pooling proposal, it strictly prefers pooling proposals to separating proposals when $\beta \geq \bar{\beta}$, which is one of the constraints that defines the case. We then confirm that the pooling proposal exists in Case 1.

$A$ prefers the pooling proposal when $U_A(x_{7L}^*) \geq U_A(x_{7H}^*)$, or

$$x_{7L}^* + \delta \frac{x_{4L}^*}{1-\delta} \geq \beta \left(-a + \delta x_{5H}^* + \delta^2 \frac{x_{1L}^*}{1-\delta}\right) + (1 - \beta) \left(x_{7H}^* + \delta \frac{x_{3H}^*}{1-\delta}\right) \iff \beta \geq \bar{\beta}.$$

Therefore, as long as the pooling proposal exists for all of Case 1, $A$ will choose it in equilibrium, whether or not the separating proposal exists. To show when it exists, recall that the most that $B$ can promise to accept in the first period is $x_{7L} = 1$, and when

$$1 + \delta \frac{x_{4L}^*}{1-\delta} \geq -a + \delta x_{6L}^* + \delta^2 \frac{x_{1L}^*}{1-\delta} \iff s \leq -1 + \frac{p \delta^2}{1 + a(-1+\delta) + \delta(p+\delta) + (-1+\delta)\delta^2 b_L},$$

there is sure to be a proposal $x_{7L}^*$, as defined above, that $A$ prefers to attacking. Finally, note that, as long as $0 < \delta < 1$, the above constraint is greater than $s$, ensuring that, for Case 1, $A$'s equilibrium strategy is to make a pooling proposal $x_{7L}^*$, which both types of $B$ accept.

**Case 2.** $\beta \geq \bar{\beta}$ and $s < s \leq \hat{s}$.

*Post-battle, pre-shift:* if facing both types, make a separating proposal; if one type, attack the strong but satisfy the weak. *Post-shift:* satisfy the strongest type present.

First, define a pooling proposal as one that both types of $B$ accept, which implies that any type’s rejection is out-of-equilibrium. This engages $A$’s out-of-equilibrium beliefs, $\beta' = 1$. The strong type accepts iff

$$1 - x_{7L} + \delta \frac{1 - x_{4L}^*}{1-\delta} \leq -b_L + \delta \left(\frac{1-p}{1-\delta} - b_L\right) \iff x_{7L} \leq 1 + \frac{\delta}{1-\delta} (p - x_{4L}^*) + (1 + \delta) b_L. \quad (9)$$

The weak type accepts iff

$$1 - x_{7L} + \delta \frac{1 - x_{4L}^*}{1-\delta} \geq -b_H + \delta \left(\frac{1-p}{1-\delta} - a\right),$$
where its payoffs differ from the strong type’s only in terms of its per-battle costs, \( b_H > b_L \).

Therefore, since the weak is worse off by rejecting, it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets

\[
x^{\ast}_{7L} = \min\{1 + \frac{\delta}{1 - \delta} (p - x^{\ast}_{4L}) + (1 + \delta) b_L, 1\}.
\]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. The incentive compatibility constraints are

\[
1 - x^{\ast}_{7H} + \frac{1 - x^{\ast}_{3H}}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) \quad (b_H)
\]

\[
-b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) > 1 - x + \delta (-b_L) + \delta^2 \frac{1 - x^{\ast}_{7H}}{1 - \delta} \quad (b_L),
\]

which reduce to

\[
1 + \delta^2 \frac{1 - x^{\ast}_{4L}}{1 - \delta} + b_L - \delta \left( \frac{1 - p}{1 - \delta} \right) < x^{\ast}_{7H} \leq 1 + \frac{\delta}{1 - \delta} (p - x^{\ast}_{3H}) + (1 + \delta) b_H.
\]

This inequality holds as long as \( b_L < b_H \), which is true by assumption. Therefore, when it wishes to separate types, \( A \) sets

\[
x^{\ast}_{7H} = \min\{1 + \frac{\delta}{1 - \delta} (p - x^{\ast}_{3H}) + (1 + \delta) b_H, 1\}.
\]

To characterize \( A \)’s best reply, we first show that, conditional on their existence, \( A \) prefers making a pooling over a separating proposal in Case 2 iff \( U_A(x^{\ast}_{7L}) \geq U_A(x^{\ast}_{7H}) \), or

\[
x^{\ast}_{7L} + \delta \frac{x^{\ast}_{4L}}{1 - \delta} \geq \beta \left( -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right) + (1 - \delta) \left( x^{\ast}_{7H} + \delta \frac{x^{\ast}_{3H}}{1 - \delta} \right) \Rightarrow \beta \geq (b_H - b_L) / (a + b_H) = \overline{\beta},
\]

which is one of the constraints that defines Case 2.

We next show \( A \) only makes pooling proposals in Case 2, because it is guaranteed to exist in this case. By the logic used above, its preference for pooling over separating proposals means that it will always choose the former in equilibrium as long as \( s \) is not so large to preclude its existence. The most that \( B \) can promise to yield to \( A \) in the first period, \( x_{7H} = 1 \), and when
A prefers attack to such a generous proposal, then there exists no \( x_{7L} \) such that \( A \) cannot be induced not to attack. Therefore, a pooling proposal exists that it prefers to attack iff

\[
1 + \delta \frac{x_{4L}}{1 - \delta} \geq -a + \delta \left[ \beta \left( \frac{p}{1 - \delta} - a \right) + (1 - \beta) \left( x_{6H}^* + \delta \frac{x_{1H}}{1 - \delta} \right) \right],
\]

or

\[
s \leq \frac{(1 - \delta)(1 + a + \delta - p\delta + a\beta\delta + (-1 + \beta)\delta b_H + \delta(1 + \delta)b_L)}{-1 + \delta(p + \delta) + a(-1 + \delta)(1 + \beta\delta) + (1 - \delta)(-1 + \beta)b_H + (1 + \delta)b_L}.
\]

The above constraint is greater than \( \hat{s} \), which defines the case, as long as \( 0 < \delta < 1 \). Therefore, in Case 2, \( A \)'s best response is to make the pooling proposal \( x_{7L}^* \), after which it does not update its beliefs.

**Case 3.** \( \beta \geq \beta \) and \( s > \hat{s} \)

*Post-battle, re-shift:* attack regardless of beliefs. *Post-shift:* satisfy the strongest type present.

Since Case 3 differs from Case 2 only in \( A \)'s payoff for attack, equilibrium separating and pooling proposals are identical to those defined in Case 2. As a result, \( A \) will prefer to make the pooling proposal over the separating proposal when \( s \) is sufficiently low that they both exist. However, the ranges over which the proposals exist differ given \( A \)'s new payoff for attack.

To determine \( A \)'s best reply, we need only define the thresholds over \( s \) below which each proposal exists. As before, a pooling proposal exists that \( A \) prefers to attack iff

\[
1 + \delta x_{4L}^* 1 - \delta \geq -a + \left( \frac{p}{1 - \delta} - a \right) \iff s \leq -1 + \frac{p\delta^2}{-1 + \delta(p + \delta) + a(-1 + \delta^2) + \delta(-1 + \delta^2)b_L} = \tilde{s}.
\]

Otherwise, when \( s > \tilde{s} \), there exists no pooling proposal that \( A \) prefers to attack. A separating proposal exists iff

\[
\beta \left[ -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right] + (1 - \beta) \left( 1 + \delta \frac{x_{3H}^*}{1 - \delta} \right) \geq -a + \left( \frac{p}{1 - \delta} - a \right),
\]

or

\[
s \leq -1 + \frac{p\delta^2}{-1 + \delta(p + \delta) + a(-1 + \delta^2) + (1 - \delta)\delta(b_H + \delta b_L)} \equiv \bar{s}. \]
Otherwise, when \( s > \bar{s} \), there exists no separating proposal that \( A \) prefers to attack.

Since \( b_H > b_L \), it is also the case that \( \bar{s} > \bar{\bar{s}} \). Therefore, we can characterize \( A \)'s best response in Case 3 in the following way. When \( \bar{s} \leq s < \bar{\bar{s}} \), both proposals exist and \( A \) makes the pooling \( x^*_L \). When \( \bar{s} \leq s < \bar{\bar{s}} \), the separating proposal exists but the pooling does not, and since \( s < \bar{s} \), \( A \) makes the separating \( x^*_H \). Finally, when \( s > \bar{s} \), \( A \) attacks.

**Case 4.** \( \underline{\beta} \leq \beta < \bar{\beta} \) and \( s \leq \bar{s} \)

*Post-battle, pre-shift:* if facing both types, make a separating proposal; if one type, satisfy it.

*Post-shift:* if both types, make a separating after one battle but a pooling proposal after no battles; if one type, satisfy it.

First, define a pooling proposal as one that both types of \( B \) accept, which implies that any type's rejection is out of equilibrium. This engages \( A \)'s out-of-equilibrium beliefs, \( \beta' = 1 \). The strong type accepts iff

\[
1 - x^*_L + \delta \frac{1 - x^*_4L}{1 - \delta} \geq -b_L + \delta(1 - x^*_5L) + \delta^2 \frac{1 - x^*_4L}{1 - \delta} \iff x^*_L \leq 1 + \delta \frac{1 - x^*_4L}{1 - \delta} - \delta(1 - x^*_5L) - \delta^2 \frac{1 - x^*_4L}{1 - \delta} + b_L.
\]

The weak type accepts such a proposal iff

\[
1 - x^*_L + \delta \frac{1 - x^*_4L}{1 - \delta} \geq -b_H + \delta(1 - x^*_5L) + \delta^2 \frac{1 - x^*_4L}{1 - \delta},
\]

and as before, the weak type's payoffs differ only in its lower payoff to rejection, which ensures that it accepts all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets

\[
x^*_L = \min\{1 + \delta \frac{1 - x^*_4L}{1 - \delta} - \delta(1 - x^*_5L) - \delta^2 \frac{1 - x^*_4L}{1 - \delta} + b_L, 1\}.
\]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. Incentive compatibility requires that
the weak accept and the strong reject, or

\[ 1 - x_{7H} + \frac{1 - x_{3H}^*}{1 - \delta} \geq -b_H + \delta(1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} \quad (b_H) \]

\[ -b_L + \delta(1 - x_{5L}^*) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} > 1 - x_{7H} + \delta(-b_L) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} \quad (b_L), \]

which reduces to

\[ 1 - \delta(1 - x_{5L}^*) + (1 - \delta)b_L < x_{7H} \leq 1 + \delta \frac{1 - x_{3H}^*}{1 - \delta} - \delta(1 - x_{5L}^*) - \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} + b_H. \]

This inequality holds, ensuring the existence of a separating proposal, when \( b_H > b_L \), which is true by construction. Therefore, when it wishes to make a proposal that only the weak type accepts, \( A \) sets

\[ x_{7H}^* = \min\{1 + \delta \frac{1 - x_{3H}^*}{1 - \delta} - \delta(1 - x_{5L}^*) - \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} + b_H, 1\}. \]

To characterize \( A \)'s best reply, we first show that, conditional on their existence, \( A \) always prefers to make a pooling proposal over a separating one in Case 4. \( A \) prefers to propose \( x_{7L}^* \) iff \( U_A(x_{7L}^*) \geq U_A(x_{7H}^*) \), or

\[ x_{7L}^* + \delta \frac{x_{4L}^*}{1 - \delta} \geq \beta \left(-a + \delta x_{5L}^* + \delta^2 \frac{x_{1L}^*}{1 - \delta}\right) + (1 - \beta) \left(x_{7H}^* + \delta \frac{x_{3H}^*}{1 - \delta}\right) \Leftrightarrow \beta \geq \beta, \]

a constraint that helps define the case. Therefore, as long as the pooling proposal exists, \( A \) will make it in equilibrium in Case 4. The most that \( B \) can promise to yield to \( A \) in the current period is \( x_{7L} = 1 \), and iff \( u_A(\text{att}_7) > u_A(x_{7L}^*) \), there exists no nosreparating proposal that \( A \) prefers to attacking. \( A \) attacks iff

\[ -a + \delta \left[ \beta \left( \frac{b}{1 - \delta} - a \right) \right] + (1 - \beta) \left(x_{6H}^* + \delta \frac{x_{2H}^*}{1 - \delta}\right) > 1 + \delta \frac{x_{4L}^*}{1 - \delta}, \]

or

\[ s > \frac{(1 - \delta)(1 + a + \delta - p\delta + a\beta\delta + (-1 + \beta)\delta b_H + \delta(1 + \delta)b_L)}{-1 + \delta(p + \delta) + a(-1 + \delta)(1 + \delta) + (-1 + \delta)\delta((-1 + \beta)b_H + (1 + \delta)b_L)}. \]

Algebra shows that, as long as \( 0 < \delta < 1 \) (true by construction), the attack constraint falls above \( s \), which ensures that the attack constraint is never satisfied in Case 4, and \( A \) will make the pooling proposal regardless of whether the separating proposal exists. Therefore, in Case 4, \( A \) makes a pooling proposal \( x_{7L}^* \), which both types of \( B \) accept.
Case 5. \( \beta \leq \beta < \beta^* \) and \( s < s \leq \hat{s} \).

Post-battle, pre-shift: if both types, make a separating proposal; if one type, attack the strong but satisfy the weak. Post-shift: if both types, make a separating after one battle but a pooling proposal after no battles; if one type, satisfy it.

First, define a pooling proposal as one that both types of \( B \) accept, which implies that any type's rejection is out of equilibrium. This engages \( A \)'s out-of-equilibrium beliefs, \( \beta' = 1 \). The strong type accepts when

\[
1 - x_7^L + \delta \frac{1 - x_4^L}{1 - \delta} \geq -b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) \Leftrightarrow x_7^L \leq 1 + \delta \frac{1 - x_4^L}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_L.
\]

The weak type also accepts when

\[
1 - x_7^L + \delta \frac{1 - x_4^L}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right).
\]

As before, the weak type's payoffs differ only in its lower payoff to rejection, which ensures that it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets

\[
x_7^* = \min \{1 + \delta \frac{1 - x_4^L}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_L, 1 \}.
\]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. The incentive compatibility constraints are

\[
1 - x_7^H + \delta \frac{1 - x_3^H}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right) \quad (b_H)
\]

\[
-b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) > 1 - x_7^H + \delta (-b_L) + \delta^2 \frac{1 - x_1^L}{1 - \delta} \quad (b_L),
\]

which reduce to

\[
1 + \delta^2 \frac{1 - x_1^L}{1 - \delta} + b_L - \delta \left( \frac{1 - p}{1 - \delta} \right) < x_7^H \leq 1 + \delta \frac{1 - x_3^H}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_H.
\]
This inequality holds, ensuring the existence of a separating proposal, when \( b_L < b_H \), which is true by construction. Therefore, to make a proposal that only the weak type accepts, \( A \) sets

\[
x^*_H = \min \{1 + \delta \left( \frac{1 - x^*_H}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) \right) + (1 + \delta) b_H, 1 \}.
\]

To characterize \( A \)'s best reply, we first determine the conditions under which \( A \) chooses to make a pooling over a separating proposal, conditional on the existence of both proposals. \( A \) does so when

\[
U_A(x^*_L) \geq U_A(x^*_H),
\]

\[
x^*_L + \delta \frac{x^*_L}{1 - \delta} \geq \beta \left( -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right) + (1 - \beta) \left( x^*_H + \delta \frac{x^*_H}{1 - \delta} \right)
\]

\[
\iff \beta \geq \frac{(1 + \delta)(b_H - b_L)}{1 + (1 + \delta)(a + b_H)} \equiv \hat{\beta}.
\]

This threshold, \( \hat{\beta} \), is guaranteed to fall in between \( \beta \) and \( \beta ' \), which define the case, as long as \( b_H > b_L \), which is true by assumption.

Next, we determine the conditions under which each proposal exists in Case 5. We begin with the pooling proposal, and as before, we compare it to the case in which \( B \) promises as much as it can in the current period, \( x^*_L = 1 \), such that \( A \) prefers attack to any pooling proposal iff

\[
-a + \delta \left[ \beta \left( \frac{p}{1 - \delta} - a \right) + (1 - \beta) \left( x^*_H + \delta \frac{x^*_H}{1 - \delta} \right) \right] > 1 + \delta \frac{x^*_L}{1 - \delta},
\]

or when

\[
s > -\frac{(1 + \delta) \left( 1 + a - p \delta + a \beta \delta + (-1 + \beta) \delta b_H + \delta (1 + \delta) b_L \right)}{1 + \delta (p + \delta) + a(-1 + \delta)(1 + \beta \delta) + (-1 + \delta) \delta \left( (-1 + \beta)(b_H + (1 + \delta) b_L) \right)}.
\]

Algebra again shows that the attack constraint above falls above \( \hat{s} \) as long as \( 0 < \delta < 1 \), which is true by construction. Therefore, there always exists a pooling proposal that \( A \) prefers to attack in Case 5.

We next compare attack to a separating proposal in which \( B \) promises as much as it can in the current period, \( x^*_H = 1 \), such that \( A \) prefers attack to any separating proposal iff

\[
-a + \delta \left[ \beta \left( \frac{p}{1 - \delta} - a \right) + (1 - \beta) \left( x^*_H + \delta \frac{x^*_H}{1 - \delta} \right) \right] > \beta \left( -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right) + (1 - \beta) \left( 1 + \delta \frac{x^*_H}{1 - \delta} \right),
\]
or when
\[ s > -1 + \frac{p\delta^2}{-1 + a(-1 + \delta) + \delta(p + \delta) + (-1 + \delta)\delta^2 b_L}. \]

Algebra again shows that the attack constraint above falls above \( \hat{s} \) as long as \( 0 < \delta < 1 \), which is true by construction, ensuring that there always exists a separating proposal that \( A \) prefers to attack in Case 5. Therefore, in Case 5, \( A \) never attacks. When \( \beta \geq \hat{\beta} \), \( A \) proposes \( x^*_L \) and both types of \( B \) accept, and when \( \beta < \hat{\beta} \), \( A \) proposes \( x^*_H \), which only the weak type accepts.

**Case 6.** \( \beta \leq \beta < \hat{\beta} \) and \( s > \hat{s} \).

*Post-battle, pre-shift:* attack regardless of beliefs. *Post-shift:* if both types, make a separating after one battle but a pooling proposal after no battles; if one type, satisfy it.

First, define a pooling proposal as one that both types of \( B \) accept, which implies that any type’s rejection is out of equilibrium. This engages \( A \)’s out-of-equilibrium beliefs, \( \beta' = 1 \). The strong type accepts when
\[
1 - x^*_L + \delta \frac{1 - x^*_L}{1 - \delta} \geq -b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right), \iff x^*_L \leq 1 + \delta \frac{1 - x^*_L}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_L.
\]

The weak type accepts when
\[
1 - x^*_L + \delta \frac{1 - x^*_L}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right).
\]

As before, the weak type’s payoffs differ only in its lower payoff to rejection, which ensures that it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets
\[
x^*_L = \min \{ 1 + \delta \frac{1 - x^*_L}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_L, 1 \}.
\]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. The incentive compatibility con-
Constraints are

\[ 1 - x_{7H} + \delta \frac{1 - x_{3H}^*}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right) \quad (b_H) \]
\[ -b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) > 1 - x_{7H} + \delta (-b_L) + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} \quad (b_L) \]

which reduce to

\[ 1 + \delta^2 \frac{1 - x_{1L}^*}{1 - \delta} + b_L - \delta \left( \frac{1 - p}{1 - \delta} \right) < x_{7H} \leq 1 + \delta \frac{1 - x_{3H}^*}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_H. \]

This inequality holds, ensuring the existence of a separating proposal, when \( b_L < b_H \), which is true by construction. Therefore, to make a proposal that only the weak type accepts, \( A \) sets

\[ x_{7H} = \min \{1 + \delta \frac{1 - x_{3H}^*}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_H, 1 \}. \]

We begin the characterization of \( A \)'s best response by noting that the same threshold divides its willingness to make separating and pooling proposals that divides it in Case 5, or \( \hat{\beta} \). (Indeed, each side's payoffs for each action and the relevant proposals are also identical across cases.) Therefore, to complete \( A \)'s strategy in Case 5, we again determine the conditions under which each proposal exists as a function of \( s \).

Begin with the pooling proposal. The most that \( B \) can promise in the current period is \( x_{7L} = 1 \), which implies that \( A \) prefers attack to any pooling proposal when

\[ -a + \delta \left( \frac{p}{1 - \delta} - a \right) > 1 + \delta \frac{x_{4L}^*}{1 - \delta} \Leftrightarrow s > \bar{s}, \]

as defined in Case 3. Otherwise, when \( s \leq \bar{s} \), a pooling proposal that \( A \) prefers to make is sure to exist. Now consider the separating proposal. The most that \( B \) can promise in the current period is \( x_{7H} = 1 \), which implies that \( A \) refers attack to any separating proposal when

\[ -a + \delta \left( \frac{p}{1 - \delta} - a \right) > \beta \left[ -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right] + (1 - \beta) \left( 1 + \delta \frac{x_{3H}^*}{1 - \delta} \right) \Leftrightarrow s > \bar{s}, \]

as defined in Case 3. Otherwise, a separating proposal that \( A \) prefers to make is sure to exist.

Therefore, the following are \( A \)'s best responses in Case 6. When \( \beta \geq \hat{\beta} \) and \( s \leq \bar{s} \), both proposals exist, and \( A \) makes the pooling \( x_{7L}^* \), which both types of \( B \) accept. When \( \beta < \hat{\beta} \) and \( s \leq \bar{s} \),
both proposals exist, and $A$ makes the separating $x^*_H$, which only the strong $B$ rejects. When $\bar{s} < s \leq \tilde{s}$, only the pooling proposal exists, which $A$ prefers to attack and therefore makes. Finally, when $s > \tilde{s}$, there exists no proposal that $A$ prefers to attack, and it attacks. $A$ only updates its beliefs after proposing $x^*_H$.

**Case 7.** $\beta < \beta_0$ and $s \leq \bar{s}$.

*Post-battle, pre-shift:* if both types, make a separating proposal; if one type, satisfy it. *Post-shift:* if both types, make a separating proposal; if one type, satisfy it.

First, define a pooling proposal as one that both types of $B$ accept, which implies that rejection is out of equilibrium. This engages $A$'s out-of-equilibrium beliefs, $\beta' = 1$. The strong type accepts when

$$1 - x^*_L + \delta (-b_L) + \delta^2 \left( \frac{1 - x^*_L}{1 - \delta} \right) \geq -b_L + \delta (1 - x^*_L) + \delta^2 \frac{1 - x^*_L}{1 - \delta} \Rightarrow x^*_L \leq 1 - \delta (b_L) + b_L - \delta (1 - x^*_L).$$

The weak type accepts when

$$1 - x^*_L + \delta (1 - x^*_H) + \delta^2 \frac{1 - x^*_H}{1 - \delta} \geq -b^*_H + \delta (1 - x^*_L) + \delta^2 \frac{1 - x^*_L}{1 - \delta},$$

or when

$$x^*_L \leq 1 + \delta (1 - x^*_H) + \delta^2 \frac{1 - x^*_H}{1 - \delta} + b^*_H - \delta (1 - x^*_L) - \delta^2 \frac{1 - x^*_L}{1 - \delta}.$$ 

The weak type's acceptance constraint falls above the strong's as long as $b^*_H > b_L$, which ensures that it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, $A$ sets

$$x^*_L = \min \{1 - \delta (b_L) + b_L - \delta (1 - x^*_L), 1\}.$$

Next, define a separating proposal as one that only the weak type of $B$ accepts, leading $A$ to believe $\beta = 1$ upon rejection and $\beta = 0$ upon acceptance. The incentive compatibility constraints
are
\[
1 - x_{7H} + \delta \frac{1 - y_{h,3}^{1}}{1 - \delta} \geq -b_{H} + \delta (1 - x_{5L}^{*}) + \delta^{2} \frac{1 - x_{1L}^{*}}{1 - \delta} \quad (b_{H})
\]
\[
-b_{L} + \delta (1 - x_{5L}^{*}) + \delta^{2} \frac{1 - x_{1L}^{*}}{1 - \delta} > 1 - x_{7H} + \delta (-b_{L}) + \delta^{2} \frac{1 - x_{1L}^{*}}{1 - \delta} \quad (b_{L}),
\]
which reduce to
\[
1 + \delta (-b_{L}) + b_{L} - \delta (1 - x_{5L}^{*}) < x_{7H} \leq 1 + \delta \frac{1 - x_{3H}^{*}}{1 - \delta} - \delta (1 - x_{5L}^{*}) - \delta^{2} \frac{1 - x_{1L}^{*}}{1 - \delta} + b_{H}.
\]
This inequality holds, ensuring the existence of a separating proposal, when \( b_{H} > b_{L} \), which is true by construction. Therefore, to make a separating proposal, \( A \) sets
\[
x_{7H}^{*} = \min\{1 + \delta \frac{1 - x_{3H}^{*}}{1 - \delta} - \delta (1 - x_{5L}^{*}) - \delta^{2} \frac{1 - x_{1L}^{*}}{1 - \delta} + b_{H}, 1\}.
\]
To characterize \( A \)'s best reply, we first show that \( A \) never chooses to make a pooling over a separating proposal when both exist. \( A \) would make the pooling proposal iff \( U_{A}(x_{7L}^{*}) \geq U_{A}(x_{7H}^{*}) \), where
\[
U_{A}(x_{7L}^{*}) = x_{7L}^{*} + \delta \left[ -a + \delta \frac{x_{1L}^{*}}{1 - \delta} \right] + (1 - \beta) \left( x_{4L}^{*} + \delta \frac{x_{3H}^{*}}{1 - \delta} \right)
\]
and
\[
U_{A}(x_{7H}^{*}) = \beta \left[ -a + \delta x_{5L}^{*} + \delta^{2} \frac{x_{1H}^{*}}{1 - \delta} \right] + (1 - \beta) \left( x_{7H}^{*} + \delta \frac{x_{3H}^{*}}{1 - \delta} \right),
\]
or iff \( \beta \geq (b_{H} - b_{L})/(1 + a + b_{H}) = \tilde{\beta} \), which cannot hold since \( \beta < \tilde{\beta} \) defines the case. Therefore, the separating proposal dominates the pooling proposal in Case 7 as long as both exist.

To show that \( A \) only makes the separating proposal in Case 7, it is only necessary to show that it exists for the whole case, which guarantees that it makes the separating proposal whether the pooling exists or not. The most generous separating proposal that \( B \) can promise to \( A \) in the current period is \( x_{7H} = 1 \), and when \( U_{A}(\text{att}_{7}) > U_{A}(x_{7H}^{*}) \), there exists no separating proposal that \( A \) prefers to attacking. Therefore, \( A \) attacks iff
\[
-a + \delta \left[ \beta \left( \frac{p}{1 - \delta} \right) + (1 - \beta) \left( x_{6H}^{*} + \frac{x_{1H}^{*}}{1 - \delta} \right) \right] > \beta \left[ -a + \delta x_{5L}^{*} + \delta^{2} \frac{x_{1L}^{*}}{1 - \delta} \right] + (1 - \beta) \left( 1 + \frac{x_{3H}^{*}}{1 - \delta} \right)
\]
or when

\[ s > \frac{(-1 + \delta)((1 + \beta(-1 + \delta)) + (-1 + \beta)(1 + (-1 + p)\delta) + \delta(\beta + \delta - \beta\delta)b_L)}{-a(1 + \beta(-1 + \delta))(1 + \delta) + (-1 + \beta)(1 + \delta(p + \delta)) + (\beta(-1 + \delta) - \delta)(-1 + \delta)b_L}. \]

Algebra shows that this constraint is greater than \( s \) (the constraint that defines the case) as long as \( 0 < \delta < 1 \), which is true by construction. Therefore, there always exists a separating proposal in Case 7 that \( A \) prefers to attack. \( A \) best response in Case 7 is thus to propose \( x^*_7 \), which the strong \( B \) rejects and the weak accepts, allowing \( A \) to update its beliefs.

**Case 8.** \( \beta < \beta_0 \) and \( s < s \leq \hat{s} \).

**Post-battle, pre-shift:** if both types, make a separating proposal; if one type, attack the strong but satisfy the weak. **Post-shift:** if both types, make a separating proposal; if one type, satisfy it.

First, define a pooling proposal as one that both types of \( B \) accept, which implies that rejection is out of equilibrium. This engages \( A \)'s out-of-equilibrium beliefs, \( \beta' = 1 \). The strong type accepts when

\[ 1 - x^*_7 + \delta(-b_L) + \delta^2 \frac{1 - x^*_1}{1 - \delta} = -b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) \Rightarrow x^*_7 \leq 1 + b_L + \delta^2 \frac{1 - x^*_1}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right). \]

The weak type accepts when

\[ 1 - x^*_7 + \delta(1 - x^*_4) + \delta^2 \frac{1 - x^*_3}{1 - \delta} = -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right), \]

or when

\[ x^*_7 \leq 1 + \delta(1 - x^*_4) + \delta^2 \frac{1 - x^*_3}{1 - \delta} + b_H - \delta \left( \frac{1 - p}{1 - \delta} - b_H \right). \]

The weak type's acceptance constraint falls above the strong's as long as \( b_H > b_L \), which ensures that it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets

\[ x^*_7 = \min\{1 + b_L + \delta^2 \frac{1 - x^*_1}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right), 1 \}. \]
Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. The incentive compatibility constraints are

\[
1 - x_{7H} + \delta \frac{1 - x^*_{3H}}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right) \quad (b_H)
\]

\[
-b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) > 1 - x_{7H} + \delta (-b_L) + \delta^2 \frac{1 - x^*_1}{1 - \delta} \quad (b_L),
\]

which reduce to

\[
1 + \delta^2 \frac{1 - x^*_1}{1 - \delta} + b_L - \delta \left( \frac{1 - p}{1 - \delta} \right) < x_{7H} \leq 1 + \delta \frac{1 - x^*_{3H}}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_H.
\]

This inequality holds, ensuring the existence of a separating proposal, when \( b_H > b_L \), which is true by construction. Therefore, to make a separating proposal, \( A \) sets

\[
x^*_7 = \min \{1 + \delta \frac{1 - x^*_{3H}}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right) + (1 + \delta) b_H, 1\}.
\]

To characterize \( A \)'s best reply, we first state the conditions under which \( A \) chooses to make a separating over a pooling proposal when both proposals exist. \( A \) does so iff \( U_A(x^*_7, h) > U_A(x^*_7, l) \), where

\[
U_A(x^*_7) = \beta \left( -a + \delta x^*_3 + \delta^2 \frac{x^*_1}{1 - \delta} \right) + (1 - \beta) \left( x^*_7 + \delta \frac{x^*_{3H}}{1 - \delta} \right)
\]

and

\[
U_A(x^*_7) = x^*_7 + \delta \left( \beta \left( -a + \delta x^*_3 + \delta^2 \frac{x^*_1}{1 - \delta} \right) + (1 - \beta) \left( x^*_7 + \delta \frac{x^*_{3H}}{1 - \delta} \right) \right),
\]

or when \( \beta < (b_H - b_L)/(1 + a - \delta + b_H) \). This constraint is strictly greater than \( \beta \), which defines the case. Therefore, \( A \) always prefers a separating to a pooling proposal in Case 8 when both exist.

To show that \( A \) makes the separating proposal in Case 8, it is only necessary to show that it exists for the whole case, which guarantees that it makes the separating proposal whether the pooling exists or not. The most that \( B \) can promise in a separating proposal is \( x_{7H} = 1 \), and
when $A$ prefers attacking even to a maximally generous proposal, then there exists no separating proposal that can prevent it from attacking. Therefore, $A$ attacks iff

$$-a + \delta \left( \beta \left( \frac{p}{1-\delta} - a \right) + (1-\beta) \left( x_{6H}^* + \delta \frac{x_{1H}^*}{1-\delta} \right) \right) > U_A(x_{7H}^*),$$

or when

$$s > -1 + \frac{p\delta^2}{-1 + a(-1 + \delta) + \delta(p + \delta) + (-1 + \delta)\delta^2 b_L}.$$

Algebra shows that this constraint falls above $\hat{s}$, which defines the case, as long as $0 < \delta < 1$, guaranteeing that in Case 8 there always exists a separating proposal that $A$ prefers to attacking. Therefore, in Case 8, $A$ proposes $x_{7H}^*$, which the strong $B$ rejects and the weak accepts, allowing $A$ to update its beliefs.

**Case 9. $\beta < \beta_-$ and $s > \hat{s}$.

*Post-battle, pre-shift:* attack regardless of beliefs. *Post-shift:* if both types, make a separating proposal; if one type, satisfy it.

First, define a pooling proposal as one that both types of $B$ accept, which implies that rejection is out of equilibrium. This engages $A$'s out-of-equilibrium beliefs, $\beta' = 1$. The strong type accepts iff

$$1 - x_{7L} + \delta(-b_L) + \delta^2 \frac{1 - x_{1L}^*}{1-\delta} \geq -b_L + \delta \left( \frac{1-p}{1-\delta} - b_L \right), \Leftrightarrow x_{7L} \leq 1 + b_L + \delta^2 \frac{1 - x_{1L}^*}{1-\delta} - \delta \left( \frac{1-p}{1-\delta} \right).$$

The weak type also accepts iff

$$1 - x_{7L} + \delta(1 - x_{4H}^*) + \delta^2 \frac{1 - x_{3H}^*}{1-\delta} \geq -b_H + \delta \left( \frac{1-p}{1-\delta} - b_H \right),$$

or when

$$x_{7L} \leq 1 + \delta(1 - x_{4H}^*) + \delta^2 \frac{1 - x_{3H}^*}{1-\delta} + b_H - \delta \left( \frac{1-p}{1-\delta} - b_H \right).$$
The weak type's acceptance constraint falls above the strong's as long as \( b_H > b_L \), which ensures that it is sure to accept all proposals also acceptable to the strong type. To make a pooling proposal that both types accept, \( A \) sets

\[
x^*_7L = \min\{1 + b_L + \delta^2 \frac{1 - x^*_1L}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right), 1\}.
\]

Next, define a separating proposal as one that only the weak type of \( B \) accepts, leading \( A \) to believe \( \beta = 1 \) upon rejection and \( \beta = 0 \) upon acceptance. The incentive compatibility constraints are

\[
1 - x^*_7H + \delta \frac{1 - x^*_3H}{1 - \delta} \geq -b_H + \delta \left( \frac{1 - p}{1 - \delta} - b_H \right) \quad (b_H)
\]

\[
-b_L + \delta \left( \frac{1 - p}{1 - \delta} - b_L \right) > 1 - x^*_7H + \delta (-b_L) + \delta^2 \frac{1 - x^*_1L}{1 - \delta} \quad (b_L),
\]

which reduce to

\[
1 + \delta^2 \frac{1 - x^*_1L}{1 - \delta} + b_L - \delta \left( \frac{1 - p}{1 - \delta} \right) < x^*_7H \leq 1 + (1 + \delta)b_H + \delta \frac{1 - x^*_3H}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right).
\]

This inequality holds, ensuring the existence of a separating proposal, when \( b_H > b_L \), which is true by construction. Therefore, to make a separating proposal, \( A \) sets

\[
x^*_7H = \min\{1 + (1 + \delta)b_H + \delta \frac{1 - x^*_3H}{1 - \delta} - \delta \left( \frac{1 - p}{1 - \delta} \right), 1\}.
\]

To characterize \( A \)'s best reply, we first demonstrate that it always prefers to make a separating over a pooling proposal in this case when both proposals exist. \( A \) prefers the separating proposal iff \( U_A(x^*_7H) > U_A(x^*_7L) \), where

\[
U_A(x^*_7H) = \beta \left[ -a + \delta \left( \frac{p}{1 - \delta} - a \right) \right] + (1 - \beta) \left( x^*_7H + \delta \frac{x^*_3H}{1 - \delta} \right)
\]

and

\[
U_A(x^*_7L) = x^*_7L + \delta \left[ \beta \left( -a + \delta \frac{x^*_1L}{1 - \delta} \right) + (1 - \beta) \left( x^*_4H + \delta \frac{x^*_3H}{1 - \delta} \right) \right],
\]

or when \( \beta < (b_H - b_L)/(1 + a - \delta + b_H) \), which is strictly greater than \( \beta \), which defines the case. So \( A \) always prefers separating to pooling proposals in Case 9 when both exist. As long as the
separating proposal exists, $A$ will always make it, regardless of the existence of the pooling proposal. Finally, we need only determine when such a separating proposal exists. The most that $B$ can promise in such a proposal is $x_{7H} = 1$, and when $A$ prefers attack to it, there exists no such deal that can prevent attack. Therefore, $A$ attacks iff

$$-a + \delta \left( \frac{p}{1-\delta} - a \right) > \beta \left[ -a + \delta \left( \frac{p}{1-\delta} - a \right) \right] + (1 - \beta) \left( 1 + \delta \frac{x_{7H}^*}{1-\delta} \right),$$

or when $s > \bar{s}$, as defined in Cases 3 and 6. Therefore, in Case 9, $A$ proposes $x_{7H}^*$, which the strong type rejects and the weak accepts, when $s \leq \bar{s}$, and it attacks when $s > \bar{s}$. $A$ updates its beliefs only after making the separating proposal.

**Comparative Statics**

**Proof of Result 4.** This is confirmed by

$$\frac{\partial s}{\partial b_H} = \frac{p(1-\delta)\delta^3}{(\delta(p+\delta) - a(1-\delta^2) - (1-\delta)\delta(b_H + \delta b_L) - 1)^2} > 0$$

and

$$\frac{\partial s}{\partial b_L} = \frac{p(1-\delta)\delta^2}{(p - a(1-\delta) + \delta - (1-\delta)\delta b_L - 1)^2} > 0.$$  

Therefore, as $b_H$ increases or $b_L$ decreases, the difference $\bar{s} - \underline{s}$ increases. □

**Proof of Result 5.** To see this, note that $\partial \bar{s}/\partial a > \partial \underline{s}/\partial a > 0$, or

$$\frac{p\delta^2(1-\delta^2)}{(\delta(p+\delta) - a(1-\delta^2) - (1-\delta)\delta(b_H + \delta b_L) - 1)^2} > \frac{p(1-\delta)\delta}{(p - a(1-\delta) + \delta - (1-\delta)\delta b_L - 1)^2} > 0.$$  

□

**Proof of Result 6.** To see this, note that $\partial \bar{s}/\partial p < \partial \underline{s}/\partial p < 0$, or

$$\frac{-(1-\delta)\delta^2(1+a+\delta + a\delta + \delta b_H + \delta^2 b_L)}{(\delta(p+\delta) - a(1-\delta^2) - (1-\delta)\delta(b_H + \delta b_L) - 1)^2} < \frac{-(1-\delta)\delta(1 + a + \delta b_L)}{(p - a(1-\delta) + \delta - (1-\delta)\delta b_L - 1)^2} < 0.$$  

□